The Acoustics of 3D Kelvin Foam Cell Geometries Based on Thermoviscous Acoustic Fluid Modelling

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Abstract

In a continuation of our previous work on the modelling of the dynamic microstructural viscous and thermal dissipation mechanisms in cylindrical fibrous porous materials, we have now extended the approach to include isotropic 3D Kelvin foam cell geometries having cylindrical strut profiles.

In particular, the Thermoviscous Acoustic Fluid finite element modelling capabilities of COMSOL are used to estimate the frequency-dependent viscous drag forces on the surface of the foam cell microstructure. Oscillatory heat transfer between the foam cell struts and the surrounding viscous fluid is included to allow for possible thermal expansion effects in the fluid. A representative elasticity matrix is estimated from the foam cell microstructure using periodic boundary conditions. These are then incorporated directly into the coupled poroelastic dynamic relations (Biot's equations) allowing the prediction of vibroacoustic performance, without the requirement of traditional pore shape approximations or the estimation of transport parameters.

The initial validations of the method are promising: in the low-frequency approximation representing essentially steady-state conditions, the method yields an exact comparison to a creeping flow CFD estimate of the Kelvin foam cell airflow resistance, and also to our analytical representation of the viscous drag forces on the Kelvin foam cell across the full frequency range.

This approach then allows the prediction of the vibroacoustic performance of the foam cell or in general other cellular material structures, using purely geometrical and constitutive material properties, helping to simplify the engineering of new material concepts for vibroacoustic applications.

Introduction

For the traditional acoustic modelling of poroelastic materials, measured or estimated transport parameters, along with the elastic properties of the solid phase of the material, are utilised within transfer matrix method (TMM) or finite element formulations of Biot's equations, in order to predict the acoustic performance.

In our work with fibrous porous materials [1, 2] , we reformulated the governing dynamic equations so that viscous dissipation arising from the relative motion between the coupled fluid and solid phases of the porous material was described by analytical expressions for dynamic viscous drag forces, and analytical expressions were also used to describe oscillatory heat transfer between the solid and fluid phases of the material. This approach was found to yield very promising results for the prediction of the acoustic performance of the fibrous material.

As part of the validation process of these analytical expressions, COMSOL thermoviscous acoustic fluid finite element modelling, based on [3], was used to investigate the range of applicability of the expressions, and the physics of the dissipation processes in the material.

This led to the realisation that the approach could be applied to a more generalised category of porous materials, where dynamic viscous dissipation, oscillatory heat transfer effects and elasticity could be predicted using finite element techniques for 3D geometries.

This paper then provides an overview of this approach, and demonstrates the applicability towards a generic 3D Kelvin foam cell example.

Dynamic Poroelastic Equation Summary

In our approach, the general governing coupled solid-fluid equations for a poroelastic material are formulated using the assumption that the volume fraction is uniform throughout the solid phase of the poroelastic structure; every plane cuts through a fraction ϕ of solid material per unit total area. It is also assumed that we do not have any pressure gradient forces at the solid-fluid interfaces inside the control volume boundary. If the contents of the control volume are held constant, pressurising the fluid applies a compressive stress or dilatation of the solid phase. Alternatively, if the control volume is held constant, applying a stress to the solid phase applies a pressure stress or dilatation to the fluid phase. This coupling between the solid structure and surrounding fluid then results in a set of stress-strain relations for the poroelastic composite, which we assume to be isotropic for the regular periodic Kelvin foam cell considered here.

Solid and Fluid Momentum Equations

If we now reconsider our previous poroelastic formulations for a fibrous porous material presented in [1, 2], and instead replace the solid fibres with the solid structure of a Kelvin foam cell, the equation of motion of the isotropic solid foam cell structure (solid momentum equation) is written as

$$\phi\rho_s\ddot{u}_i = \left(\frac{\partial\sigma_{ii}}{\partial x_i}+\frac{\partial\sigma_{ij}}{\partial x_j}+\frac{\partial\sigma_{ik}}{\partial x_k}\right)-F_{D_i}, \qquad (1)$$

where F_{D_i} are the components of the volume-averaged dynamic viscous drag force vector $\mathbf{F}_D$, which the elastic foam structure exerts on the fluid per unit volume, resulting from the relative motion between the solid and fluid phases of the poroelastic material. The dynamic viscous drag force is a linear function of the solid strains and fluid displacements, which for the work presented in this paper is estimated from 3D finite element simulations of the Kelvin foam cell oscillating within a surrounding thermoviscous acoustic fluid. The viscous drag force vector may also be estimated using analytical relations for simplified cylindrical geometries [1, 2], as was the case for our previous work on fibrous poroelastic materials. Note that the dynamic viscous drag force impedance vector may be defined for not only isotropic, but also fully anisotropic materials, as well as for a statistical distribution of porous cell geometries.

Likewise, a momentum balance on the fluid phase of the poroelastic Kelvin cell foam material allows the equation of motion of the viscous fluid (fluid momentum equation) to be defined as

$$(1-\phi)\rho_f\ddot{U}_i = \frac{\partial\sigma}{\partial x_i}+F_{D_i} . \qquad (2)$$

Non-Equilibrium Fluid Dilatation

Under equilibrium conditions, stress and strain for any porous material are related by coupled constitutive stress-strain relations of the form

$$\{\sigma\}=[C]\{\varepsilon\}, \qquad (3)$$

where [C] is the matrix of elastic coefficients for the solid phase of the material. When waves propagate through the poroelastic material, we can assume oscillatory heat transfer between the solid phase and the surrounding viscous fluid, resulting in a thermal expansion of the fluid phase. This leads to an extension of the fluid dilatation terms of the stress-strain relations away from equilibrium. Making use of the linearized entropy relations for a two-phase porous material, we can derive a new fluid dilatation expression [1, 2] assuming in this case isotropy, which is valid for non-equilibrium conditions

$$\chi\sigma = R\varepsilon + Q\left(e_{xx}+e_{yy}+e_{zz}\right) . \qquad (4)$$

Here, the fluid pressure term is now scaled by the frequency-dependent thermal coefficient χ

$$\chi=\left[1-\frac{\alpha\eta R}{j\omega\rho_f Cp_f(1-\phi)}\right], \qquad (5)$$

where

$$\alpha=\eta T'_f\left\{\frac{\rho_f Cp_f(1-\phi)}{\bar{Y}_e}+\frac{1}{j\omega}\right\}^{-1}, \qquad (6)$$

and $\bar{Y}_e$ is the effective thermal impedance function,

$$\bar{Y}_e=\frac{\phi}{A}\left[\frac{1}{Y_f}+\frac{1}{Y_s}\right]^{-1}, \qquad (7)$$

which is derived from the oscillatory thermal fields of the respective fluid and solid phases, Y_f and Y_s [1, 4].

Therefore, in a completely generalized way, we then assume that the dynamic behavior of any poroelastic material is defined in terms of its coupled solid and fluid momentum equations, with elasticity and dynamic viscous drag force inputs, and the non-equilibrium fluid dilatation equation, with oscillatory heat transfer inputs, as shown in Fig. (1).

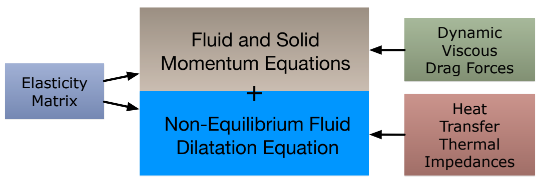

Figure 1. Dynamic poroelastic relations with elasticity, viscous drag force and oscillatory heat transfer inputs.

The determination of these inputs for a representative 3D Kelvin foam cell, using finite element simulations of periodic elasticity, thermoviscous acoustics and oscillatory heat transfer within the Structural Mechanics, Thermoviscous Acoustics and Heat Transfer in Solids and Fluids modules respectively of COMSOL [5], is presented in the following sections of this paper.

It is important to highlight that although this work focusses on the virtual numerical estimation of the inputs for the dynamic poroelastic equations, any combination of experimental, analytical or numerical estimation may be used.

Unit Kelvin Foam Cell

For this development, a simple isotropic, Kelvin-like foam cell structure was chosen, having a cell size of 3 mm, and with cylindrical struts having a diameter of 0.5 mm, as shown in Fig. (2). The foam cell has a volume of 6.55 mm^3, resulting in a cell porosity of 0.8788. Kelvin foam cells having various strut profiles [7], and completely general foam geometries and strut profiles [8] may also be considered. In addition, the approach is also well-suited to regular lattice-type structures.

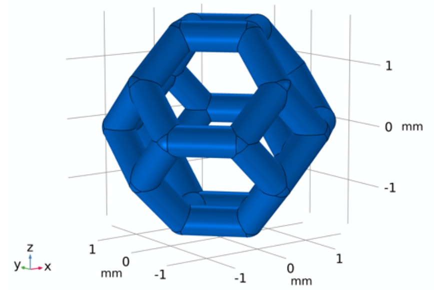

Figure 2. 3mm Kelvin foam cell, having 0.5mm diameter cylindrical struts.

The 3mm Kelvin foam cell was 3D printed as part of a periodic array of cells as shown in Fig. (3), using a PLA (polylactic acid) polymer filament material. The filament was assumed to have a density of 1235 kg/m^3, a bulk modulus of 21.1 MPa, and a Poisson's ratio of 0.3. The thermal conductivity of the PLA filament was assumed to be 0.16 W/mK at 20 deg. C, and the specific heat was assumed to be 1600 J/kgK. The properties of the viscous fluid surrounding the foam cell were that of air at 20 deg. C. It is important to note that the bulk modulus, thermal properties and actual precise geometry of the Kelvin foam cells may be modified by thermal residual stresses present in the 3D printing process, so these values should always be considered as approximate.

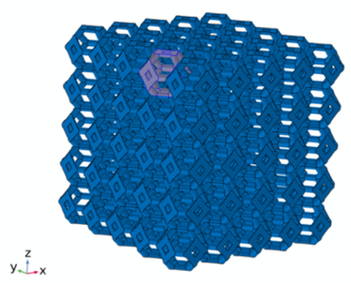

Figure 3. Kelvin foam cell periodicity.

Periodic Elasticity

In order to estimate the 6x6 elasticity matrix [C] of the 3mm Kelvin foam cell, the Structural Mechanics Module of COMSOL was used. Periodicity conditions were applied on alternating faces through a choice of work-plane cuts, and six different load cases were considered in order to estimate the elasticity matrix. A total of 475000 quadratic tetrahedral finite elements were used in the analysis, guaranteeing convergence.

The Kelvin foam cell has cubic symmetry, and using the procedure outlined in [6], the respective effective longitudinal Young's modulus and Poisson's ratio can be estimated using the following relations

$$E_L = \left(C_{11}^{\;2} + C_{11}C_{12} - 2C_{12}^{\;2}\right) / \left(C_{11} + C_{12}\right) \,, \quad (8)$$

$$\upsilon_L = C_{12} / \left(C_{11} + C_{12}\right) \,, \quad (9)$$

resulting in an estimated effective isotropic Young's modulus of 3.4 MPa, a Poisson's ratio of 0.35, and a shear modulus 1.25 MPa.

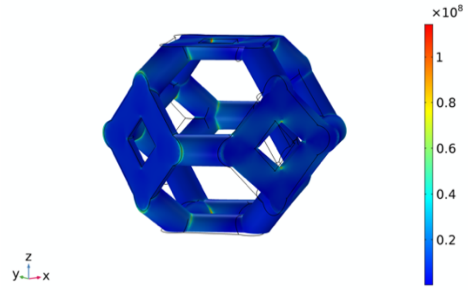

Figure 4. Typical Kelvin foam cell von Mises stress, N/m^2, for the first loading case.

Dynamic Viscous Drag Forces

In our previous work [1, 2] on the dynamics of lightweight fibrous porous materials having cylindrically-shaped fibres, we demonstrated that 2D thermoviscous acoustic fluid finite element models may be used to accurately estimate the dynamic viscous drag forces on the surface of a solid embedded within a surrounding viscous fluid, allowing the dynamic drag forces on the solid surface to be estimated, providing improved understanding of the dissipation within the boundary layer region near the solid fibre surfaces.

We now extend this work into 3D with the Kelvin foam cell application, where we consider an individual foam cell to be embedded within a surrounding spherical thermoviscous acoustic fluid layer (TVA), which is then encased within an acoustic fluid having a non-reflecting radiation outer boundary condition, as shown in Fig. (5). The radial dimensions of the TVA layer are chosen to allow sufficient decay of the viscous and thermal fields within the layer, varying according to excitation frequency.

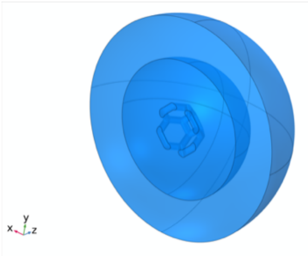

Figure 5. Sinusoidally oscillating 3 mm Kelvin foam cell embedded within a thermoviscous acoustic fluid, surrounded by an acoustic pressure field.

For a layer of actual 3D printed material, the foam cell is assumed to be repeated throughout the sample, so the estimation of the viscous behavior of a single foam cell was deemed to be sufficient. In addition, the elastic coupling between the foam cell structure, and the surrounding viscous fluid was neglected in the analysis, due to the relatively large difference between the bulk modulus of the foam cell as compared to the surrounding air.

This analysis was carried out within COMSOL, using the Thermoviscous Acoustics fluid module to represent the TVA region, and the Acoustics module to represent the encapsulating pressure fluid. The resulting finite element model consisted of quadratic tetrahedral elements for both the velocity and thermal fields within the TVA region, and linear pressure elements were used in the acoustic fluid. The number of elements used in the modelled scaled as the domain sizes varied with excitation frequency, and is not reported here. Plane symmetry conditions were used in the modelling to take advantage of the cubic symmetry of the foam cell.

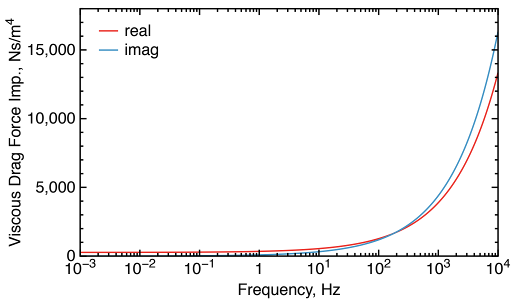

Figure 6. Real and imaginary parts of the estimated dynamic viscous drag force impedance function, Ns/m^4.

Upon application of a 0.01 micrometer harmonic excitation to the foam cell, the reaction forces on the solid surfaces of the cell were used to estimate the dynamic viscous drag forces per unit volume, for a frequency range of 0.001 - 10000 Hz, as shown in Fig. (6). Above 200 Hz, the viscous drag force behavior of the foam cell becomes inertial, while at the lowest frequencies, the real (dissipative) part asymptotes, allowing for an estimate of the static airflow resistivity of the foam cell as 261 Ns/m^4, which compares almost identically to the CFD creeping flow analysis result of 262 Ns/m^4, estimated as shown in Fig. (7).

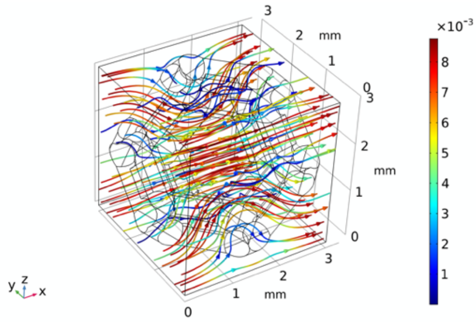

Figure 7. Velocity field streamlines of the 3 mm Kelvin foam cell, m/s.

Our future work will include a more detailed comparison with static airflow resistivity measurements, and an analytical estimate of the dynamic viscous drag forces of this idealized Kelvin foam cell having cylindrical strut profiles.

For visualisation purposes, the velocity and viscous power dissipation density fields in the vicinity of the foam cell surface are shown in Figs. (8, 9) for an excitation frequency of 1000 Hz. As we observed in our previous work on porous fibrous materials, the velocity field and the viscous power dissipation density fields, are especially concentrated in the vicinity of the solid surfaces, becoming even more concentrated within the solid surface region with increasing frequency.

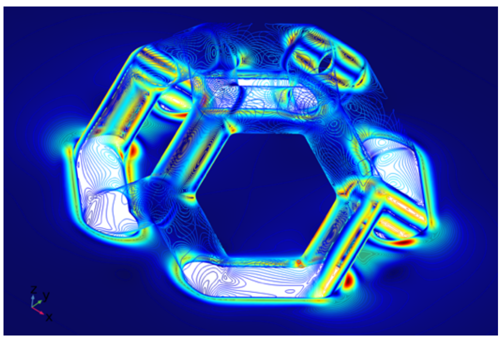

Figure 8. Velocity field in the vicinity of the 3 mm Kelvin foam cell, 1000 Hz excitation frequency.

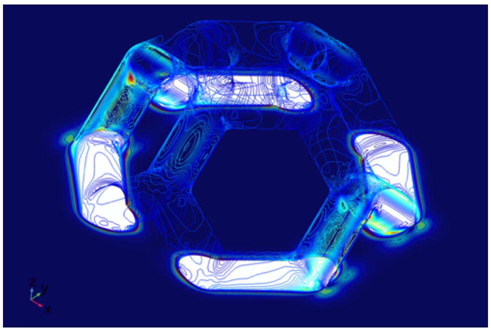

Figure 9. Viscous power dissipation density in the vicinity of the 3 mm Kelvin foam cell, 1000 Hz excitation frequency.

Solid - Fluid Oscillatory Heat Transfer

Within the TVA region, the dissipative effects of the thermal field boundary layer are negligible as compared to viscous field dissipation [1, 2], meaning that the heat transfer from the Kelvin foam cell solid structure, to the surrounding viscous fluid is responsible for a thermal expansion of the fluid only. For our previous work on highly porous glassfibre materials, this effect was significant as it resulted in an increase of the phase speed of the propagating fluid dilatational wave, effectively stiffening the fibrous material acoustically in the auditory frequency range.

For the current case of the polymer Kelvin foam cell, the thermal conductivity of the solid material is more comparable to that of the surrounding air, and the volume fraction of solid material is also much greater. This suggests that heat transfer effects in the polymer Kelvin foam cell may be of less significance than for the glassfibre insulation case studied previously.

To investigate this, the Heat Transfer in Solids and Fluids Module within COMSOL was utilised to study the coupled thermal fields within both the solid structure of the foam cell, and the surrounding air. Similar to the previous viscous drag analysis, the 3mm Kelvin foam cell was encapsulated within a thermally insulated spherical air domain, as shown in Fig. (5), with a harmonic temperature perturbation of 293.15 K applied to the foam cell surface, i.e. the solid-fluid interface region. A total of 1750000 quadratic tetrahedral thermal finite elements were used in the resulting symmetric finite element model.

The total net heat rates for both the solid structure of the foam cell, along with the surrounding air, were estimated along the surface region of the solid-fluid interface, allowing the respective solid and fluid dynamic thermal impedances to be estimated, as shown in Fig. (10). The relatively low thermal conductivity value of the polymer Kelvin foam cell, along with its large volume fraction of solid material, means that oscillatory heat transfer effects within the structure of the foam cell are important at the lower frequencies only, when its thermal impedance is less than that of the surrounding air, as shown in Fig. (10).

This suggests that for this Kelvin foam cell configuration, a first approximation would be to neglect oscillatory heat transfer effects within the solid material as an initial approximation in a poroelastic wave propagation analysis.

If the cross-sectional dimensions of the foam cell struts were significantly reduced for example, the thermal field in the solid structure would remain significant within the audible frequency range, and would need to be considered in the subsequent poroelastic wave propagation analysis.

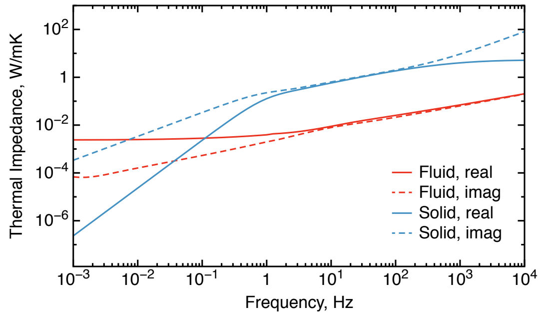

Figure 10. Real and imaginary parts of the air and foam solid thermal impedance, W/mK.

In addition, from the estimated thermal impedances, and Eqs. (6, 7), the χ coefficient can also be estimated, refer to Fig. (11), indicating a transition from isothermal to adiabatic thermal behavior at very low frequencies.

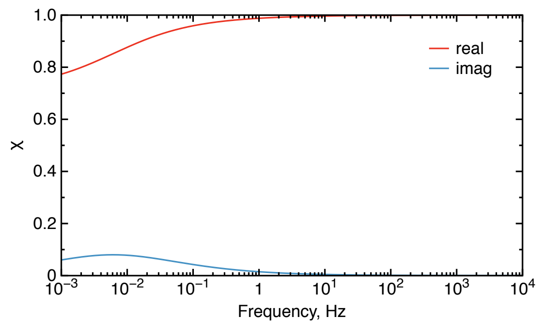

Figure 11. Real and imaginary parts of the χ coefficient, indicating the low frequency transition from isothermal to adiabatic thermal behaviour.

For reference, thermal field contours are also shown in Figs. (12, 13) for both the Kelvin foam cell structure and surrounding air, for perturbation frequencies of 1 Hz and 1000 Hz, respectively.

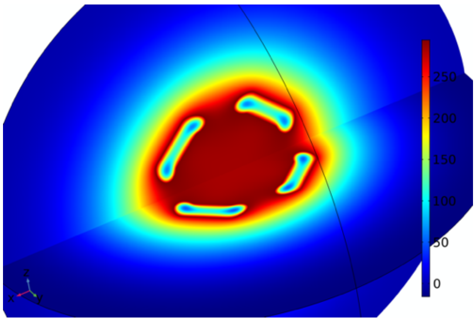

Figure 12. Slice view of the thermal fields within the 3mm Kelvin foam cell solid structure, and the surrounding air in values of K, for an excitation frequency of 1 Hz.

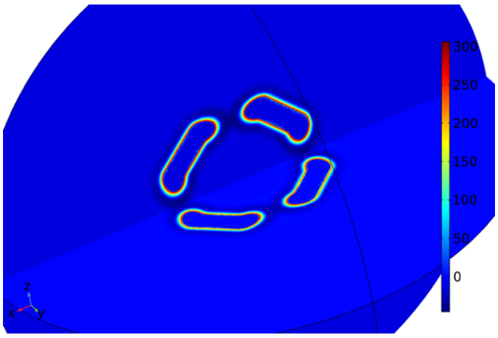

Figure 13. Slice view of the thermal fields within the 3mm Kelvin foam cell solid structure, and the surrounding air in values of K, for an excitation frequency of 1000 Hz.

Estimated Acoustic Performance

In a final step, the aforementioned estimated foam cell elasticity matrix, dynamic viscous drag forces and dynamic fluid and solid thermal impedances may be implemented in a Transfer Matrix (TMM) solution of the governing momentum and fluid dilatation equations [1], in order to predict coupled acoustic and structural wave propagation through a layer of the Kelvin foam cells.

This approach then allows us to take any porous cell geometry, determine elastic, viscous and thermal effects using finite element methods, and virtually assess the resulting acoustic performance.

For the case of the 3mm Kelvin foam cell, we have considered a 50mm thickness of periodic cells, and then considered plane acoustic wave propagation through a rigidly-backed finite thickness of the material, i.e. the standard acoustic impedance tube experiment. The normal-incidence absorption coefficient prediction for the 50 mm thickness of material is then shown in Fig. (14).

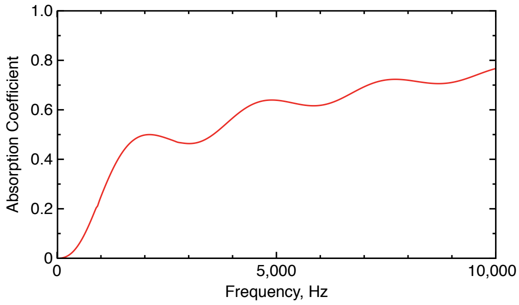

Figure 14. Estimated normal-incidence absorption coefficient for a 50 mm thickness sample of the Kelvin foam cell array, simulated using TM methods .

Conclusions

In this paper, we are presenting the early stages of our development of a general methodology for the virtual development of porous cellular materials for vibroacoustics.

The approach is based on the utilisation of high-fidelity finite element simulations to determine the dynamic viscous drag forces, oscillatory heat transfer thermal impedances and elasticity matrices of 3D porous microstructures, for input into a governing set of coupled fluid and solid momentum equations, and a modified fluid dilatation term. Subsequent transfer matrix or finite element solutions of these governing equations then allows the vibroacoustic performance of the porous cellular structure to be predicted.

We have used a 3D printed periodic Kelvin foam cell geometry to demonstrate the methodology, but more general foam geometries, including those with membranes or partially-closed cells, could also be considered. The approach would also be suitable for lattice structures as well.

Our immediate future efforts will explore additional experimental validation, and the development of improved understanding of the limits of the applicability of this approach.

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