Growth of Transient Quantum Mechanical Dirac Wave Functions Thru electric or Magnetic Fields

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Introduction: Find the relativistic quantum mechanics steady state wave function $\Psi_m(x,y,z,t)$ as a solution to the Dirac equations with pre-existing magnetic and electric potentials $\bar{A}$, ϕ. The probability density, ρ, of a particle's location is given by $\rho=\sum|\Psi_m|^2$ m=1..4

Computational Method: The EM Dirac equations [1] for the behavior of a particle of mass *m* with $M=mc/\hbar$, c=light speed, $\hbar$=Planck's constant, $\bar{\mathbf{A}}=\bar{A}e/\hbar$, $\Phi=e\phi/c\hbar$, e=charge: are solved with

$$\frac{1}{c}\frac{\partial\Psi_1}{\partial t}+\frac{\partial\Psi_4}{\partial x}-i\frac{\partial\Psi_4}{\partial y}+\frac{\partial\Psi_3}{\partial z}+i\Psi_1(\Phi+M)+i(i\mathbf{A}_y\Psi_4-\mathbf{A}_x\Psi_4-\mathbf{A}_z\Psi_3)=0$$

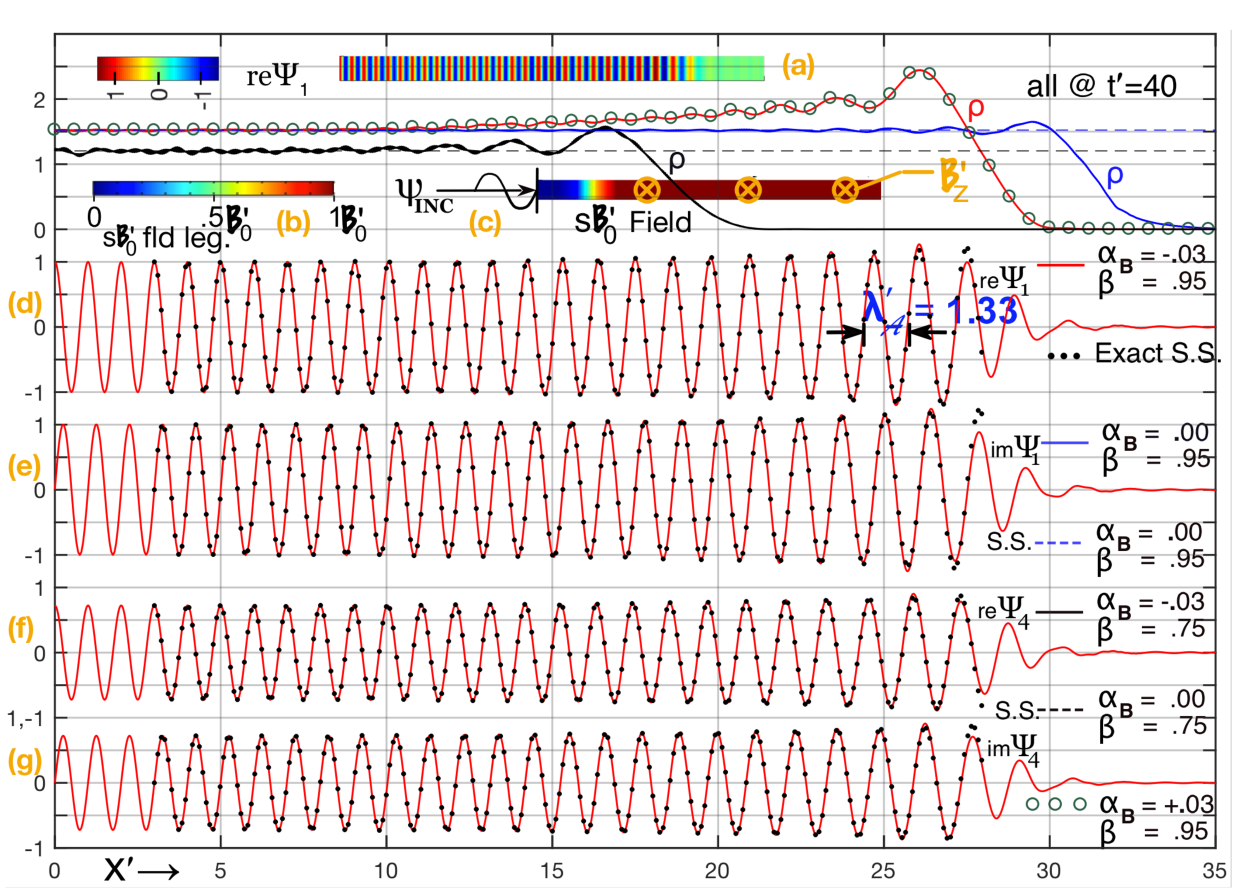
$$\frac{1}{c}\frac{\partial\Psi_2}{\partial t}+\frac{\partial\Psi_3}{\partial x}+i\frac{\partial\Psi_3}{\partial y}-\frac{\partial\Psi_4}{\partial z}+i\Psi_2(\Phi+M)+i(\mathbf{A}_z\Psi_4-\mathbf{A}_x\Psi_3-i\mathbf{A}_y\Psi_3)=0 \quad (1)$$

$$\frac{1}{c}\frac{\partial\Psi_3}{\partial t}+\frac{\partial\Psi_2}{\partial x}-i\frac{\partial\Psi_2}{\partial y}+\frac{\partial\Psi_1}{\partial z}+i\Psi_3(\Phi-M)+i(i\mathbf{A}_y\Psi_2-\mathbf{A}_x\Psi_2-\mathbf{A}_z\Psi_1)=0$$

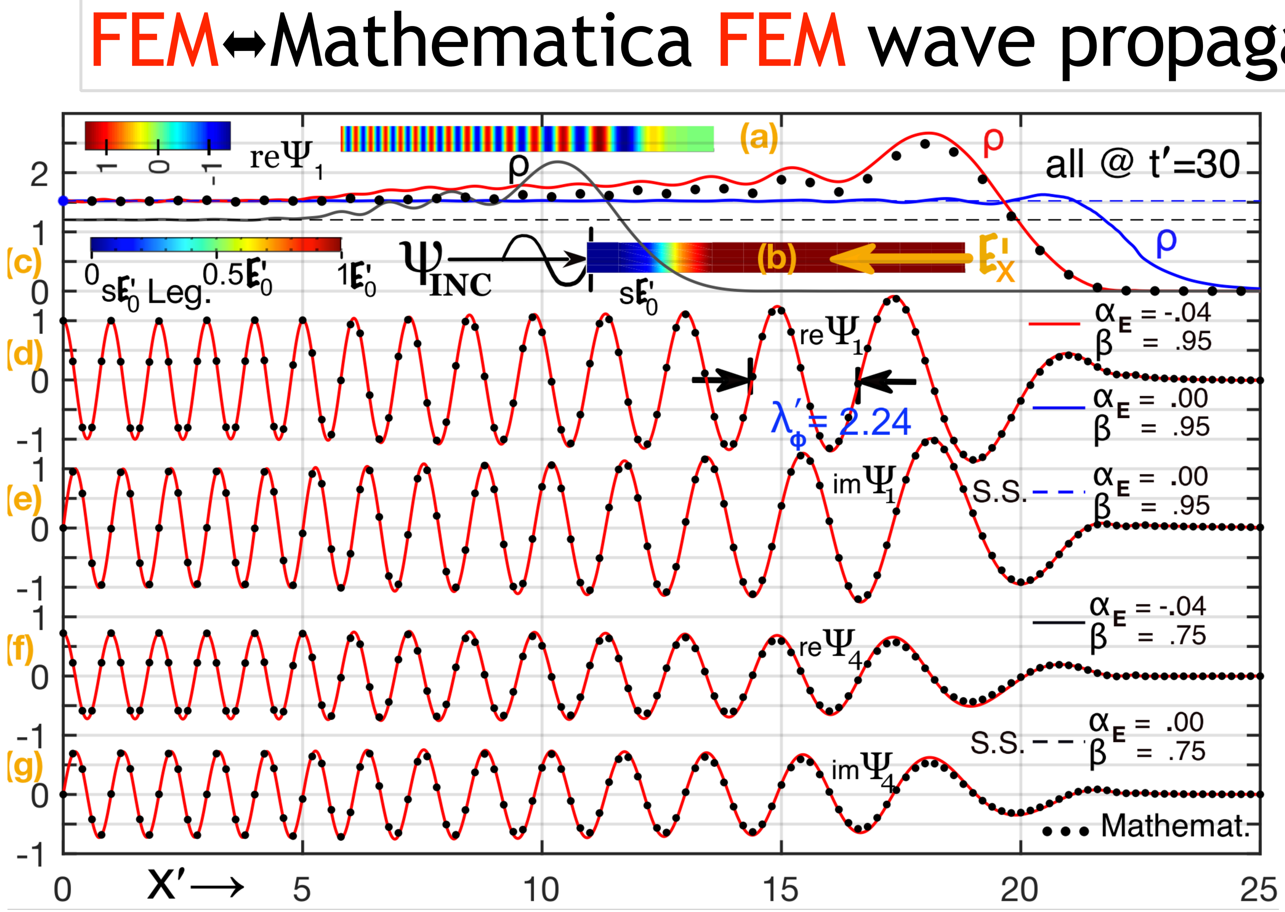
$$\frac{1}{c}\frac{\partial\Psi_4}{\partial t}+\frac{\partial\Psi_1}{\partial x}+i\frac{\partial\Psi_1}{\partial y}-\frac{\partial\Psi_2}{\partial z}+i\Psi_4(\Phi-M)+i(\mathbf{A}_z\Psi_2-\mathbf{A}_x\Psi_1-i\mathbf{A}_y\Psi_1)=0$$

COMSOL'S *"General-Form PDE"*. When the wave vector $\bar{\mathbf{k}}$ is in the xy plane, $\partial\Psi_m/\partial z$ terms drop out and the 1st & 4th eqs. decouple, where Ψ_1,Ψ_4 are solved alone.

Results: • **Fig.1** <u>PW in Magnetic $\mathcal{B}$ Field</u> below validates the $\Psi_n=\Psi_{on}\, e^{-i\omega' t'}$ end driven *Wave Guide* COMSOL FEM↔Mathematica ***Exact*** propagation vs $x'=x/\lambda_D$ and is shown for 3 values of magnetic field strength parameter α_B={.0, -0.03,+0.03}.The magnetic $\mathcal{B}'$ field effect gradually increases the λ'_A spatial wave length and ρ probability density vs +x'.



• **Fig.2** <u>PW in Electric $\mathcal{E}'$ Field</u> below validates the $\Psi_n=\Psi_{on}\, e^{-i\omega' t'}$ end driven *Wave Guide* PW COMSOL FEM↔Mathematica FEM wave propagation vs $x'=x/\lambda_D$ and is shown for 2 values of electric field strength parameter α_E={.0, -0.04}. The electric $\mathcal{E}'$ field effect gradually increases the λ'_ϕ spatial wave length and ρ probability density vs +x' . Step $\int$ shaped rise functions ≡ s .



• **Fig.3** <u>CYL.Wave in Electric $\mathcal{E}'$ Field</u> upper right validates the $\Psi_n=\Psi_{on}(\theta)\, e^{-i\omega' t'}$ inner radius driven *cylindrical wave* COMSOL FEM↔EXACT wave propagation vs x',y' and is shown for 2 values of electric field strength parameter α_E={.0, -0.04}. Figs.(3a-b) compare Exact reΨ_4 S.S. limit vs FEM @ $t'=t/T_D=12$ for $\mathcal{E}'$ field off (i.e. $\alpha_E=0$).

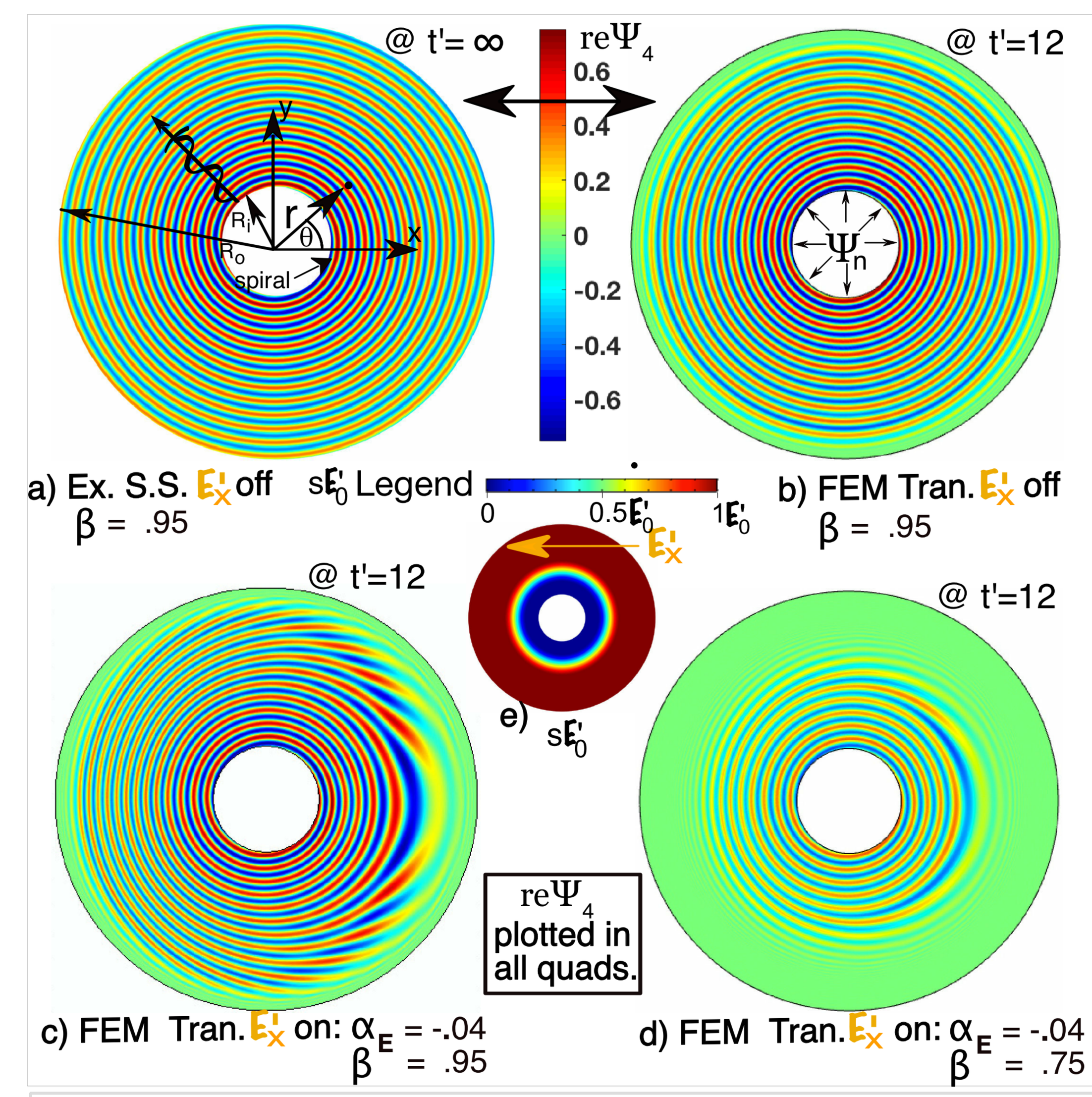
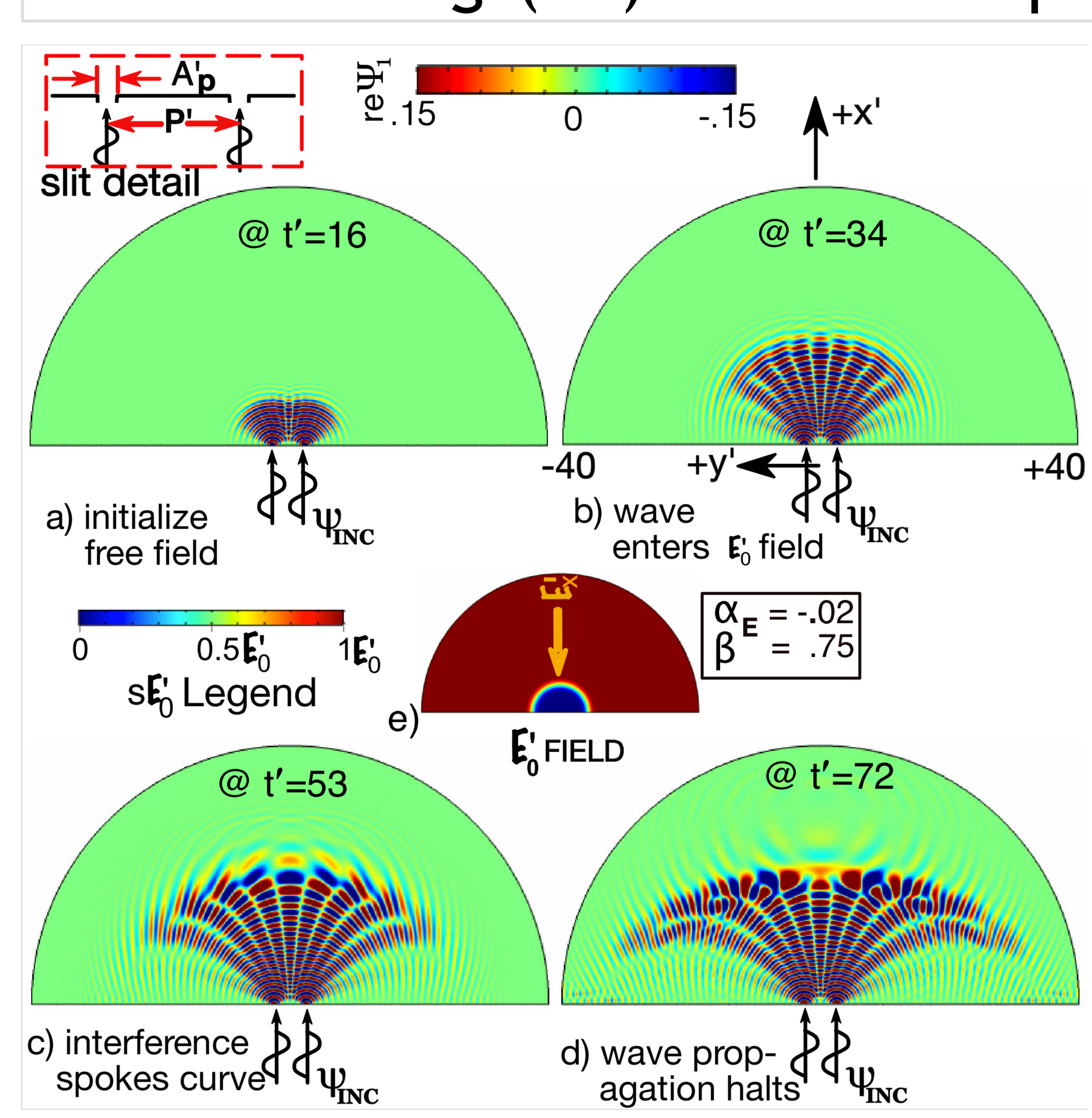


Fig.(3c) shows the effect of E_x' turned on where along x', the Ψ_4 wavelength gradually expands opposing E_x' @ θ=0° & compresses in-line with E_x' @ θ=180° while passing thru the $\mathcal{E}$ field. Fig.(3d) shows the effect of a different β=.75 frequency parameter.

• **Fig.4** <u>2 Slit Demo; Electric $\mathcal{E}'$ Field *On*</u> Particles fired at 2 slits, is a classic quantum mechanics demo, represented by a *free field* $\Psi_n=\Psi_{on}\, e^{-i(x'k'_D-\omega' t')}$ PW wave function incident upon the slits. Figs.(4a-d) show the time step transient growth of the reΨ_1 component. Classical bands of constructive interference form while the E_x' field is on except for two differences. The effect of the E_x' field (with electric field strength parameter α_E=-0.02) is to: (1) curve the blade like Fig.(4c) bands compared to otherwise straight bands with the E_x' field turned off; (2) eventually halt progress of the outgoing radial propagation (e.g. not much radial wavefront movement between snapshot Fig.(4c)@t'=53 & Fig.(4d)@t'=72.



Conclusions: The *General-Form PDE* option successfully validated the EM transient Dirac Eqs. PW and CW wave solutions that resulted in growing spatial frequency and amplitude traveling waves. The classic 2 slit model produced EM influenced curved constructive interference bands and in some cases halted the forward progress of wave fronts.

References:1. P. Strange, Relativistic <u>Quantum Mech.</u>, Camb. Univ. Press 1998