Design Optimization of a Meso-scale Coriolis Vibratory Gyroscope

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Abstract

Coriolis Vibratory Gyroscopes (CVGs) are high performance devices used to measure rotation rates with very high accuracy. The vibratory element, called the resonator, is designed to possess a pair of orthogonal degenerate modes with very high Quality factors (Q-factors). The frequencies associated with these modes are required to be far separated from those of other modes. In this work, the thickness of a CVG with a disc resonator is optimized to fulfill these requirements. Modal analysis is performed to locate regimes of thicknesses so as to avoid cross coupling of modes. Further thermo elastic damping (TED) and anchor losses are estimated to predict the Q-fator of the modes of interest. The basic configuration is arrived considering sufficient separation of the functional mode from other modes. The TED based Q-factor is improved by an order through proper selection of beam thickness. The TED is found highly sensitive to thin conductive film coatings. Sensitivity of anchor loss to the geometry is also studied using the perfectly matched layer concept in Comsol.

Introduction

Gyroscopes are instruments which are used to measure angular motion. Their traditional market is for navigation, guidance and stabilization of space applications and military systems. In recent years, with the advancement of meso- and micro-electro mechanical systems, there has been an increasing demand to manufacture less expensive gyroscopes and accelerometers while maintaining a good performance[1]. In recent years, a number of practical gyros have been conceived in which the vibrations of a a flexible structure such as a rod, beam or hemisphere,is utlized to sense rotation rates. Such gyros utlize the vibrations induced by Coriolis force to sense rotations and hence come under the category of Coriolis Vibratory Gyroscopes (CVGs).

These devices overcome a major problem of rotating-wheel gyros: the need for bearings or the like to permit high-speed rotation of the gyro wheel relative to the case. The absence of bearings in vibrating-member gyros eliminates wear and errors associated with wear that are inherent in rotating wheel gyros. The absence of bearing friction in vibrating member gyros also eliminates the principal cause of power consumption. Vibrating-member gyros have other advantages. They are capable of accepting virtually unlimited angular velocity inputs without design compromises that reduce the accuracy of rotating-wheel gyros. Thus, they are ideally suited to application in strapdown inertial navigation systems. Moreover, vibrating-member gyros, in principle, are much less sensitive to linear accelerations than rotatingwheel gyros and are thus better suited to high acceleration environments than the latter. But, as with all instruments, the vibrating-member gyro is not without practical problems. In particular, an instrument that uses Foucault's principle requires an ideal vibrating member; one in which the amplitude of oscillation is not reduced by energy losses and in which there is complete dynamic symmetry so that the plane of oscillation precesses at a rate determined only by the angular rotation of the instrument. No practical instrument will be perfectly symmetrical and the mechanical energy of vibration will dissipate unless a drive system is provided to sustain the oscillation[2].

The disk resonator gyroscope (DRG) is a meso-scale, high performance, compact and planar new generation Coriolis vibratory gyroscope which is suitable for space applications. Angular rate measurement utilizes the precession of a pair of degenerate modes of a perfectly balanced (axisymmetric) DRG resonator under the influence of the Coriolis force. This pair of operating modes is usually referred as the n=2 pair of modes and each one of these is spatially separated from the other by an angle of 45 degrees. This paper discusses how the thickness of the quartz resonator is finalized, based on results of simulations performed on COMSOL 5.3.

Working principle

A body of mass m translating with a velocity $\vec{v}$, when rotated at an angular rate $\vec{\Omega}$ experiences a force equal to $2m\vec{\Omega} \times \vec{v}$. This force is in a direction perpendicular to both the axis of rotation and the direction of translation. In other words, a point mass (constrained by springs in both X and Y directions) which is set to vibrate in the X direction when rotated about the Z axis experiences vibrations in the Y direction. For a constant rotation rate, the vibrations forced in the X axis therefore gets transferred to the Y axis at the same frequency and the point mass traces a pattern in time which is the vector sum of the vibrations in the X and Y directions. For an observer rotating along with the axis (at the same rotation rate), the motion in the Y direction is a measure of the rotation rate. It is advantageous for CVG resonators to have equal stiffnesses in the X (the drive) and the Y (the sense) directions so that resonant oscillations in one direction when coupled due to Coriolis force to the other, resonates the other second mode as well. For this purpose a pair of degenerate modes (with equal frequencies but spatially orthogonal) are selected. The disc resonator is one such structure which has a pair of degenerate modes. Another requirement is that the resonator damping levels be as low as possible. This ensures a high quality factor (Q-factor) and a large amplitude for resonant vibrations. The large amplitude vibrations in the linear zone imparts more sensitivity to the gyroscope.

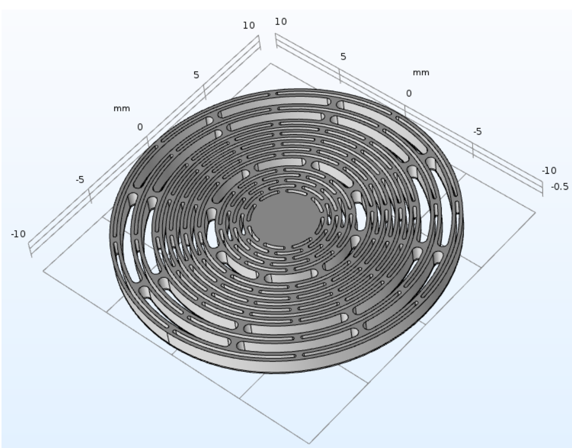

Figure 1: The three dimensional model of the DRG resonator

Modeling and Analysis method

The general modeling procedure undertaken for each of the studies performed in this work is as follows. The two dimensional sectional view of the DRG resonator was initially prepared in AutoCAD. The sketch was imported into COMSOL and extruded in the thickness direction to create the solid model. Once the solid model was created the material properties were assigned. Thereafter, the required loads were applied and the boundary conditions were esatbilshed. The solid model was meshed and solved, and the required results were plotted. The above described generalized procedure was employed for each of the following studies. Specific details are described in the corresponding sections.

A three dimensional view of the DRG is presented in Figure 1. The overall diameter of the resonator is approximately 20mm. The following material properties were used.

Material	Young's modulus	Poisson's ratio	Density
Quartz	72 GPa	0.17	2203
Aluminum	70 GPa	0.30	2700
Gold	79 GPa	0.42	1930

Table 1: Material properties

Natural frequency and mode shapes

The behavior of every structure in response to periodic dynamic loads can be understood by studying its natural frequencies and the associated mode shapes. The DRG resonator is designed to operate under sinusoidal electrostatic forcing at one of its natural frequencies. At this frequency it is required that only the mode of interest is excited. Therefore the natural frequencies of all the other modes need to be well separated from that of the working mode. The most common operating mode for the DRG resonator is the n=2 in-plane mode. This mode exists as a degenerate pair, the mode shape of each of which has its anti nodal axis spatially separated from that of the other by 45 degrees. The mode shapes of the n=2 mode is presented in Figure 2. Note that, as evident in the elevation views presented in Figure 2, these mode shapes are purely in-plane.

The natural frequencies and the mode shapes for one particular thickness value are extracted by conducting a modal analysis using an Eigenfrequency study. The study is repeated for a range of thickness values using the parametric sweep feature. The natural frequencies obtained from the Eigenfrequency study is plotted against the extrusion thickness to obtain a locus and is presented in figure 3. The red rectangular box marks the region around the n=2 mode. The frequencies around this thickness is magnified in figure 4(a) and figure 4(b). It is interesting to note that the frequency of the n=2 rises very gradually as the thickness is increased. Several frequency crossings are observed where the locus of one frequency interferes with the other. These thickness values are avoided to prevent coupling of the crossing modes. A representative plot of the coupled mode shape is presented in figure 5. Coupling destroys the perfect in-plane vibratory nature of the mode and brings in undesirable out-of-plane bending.

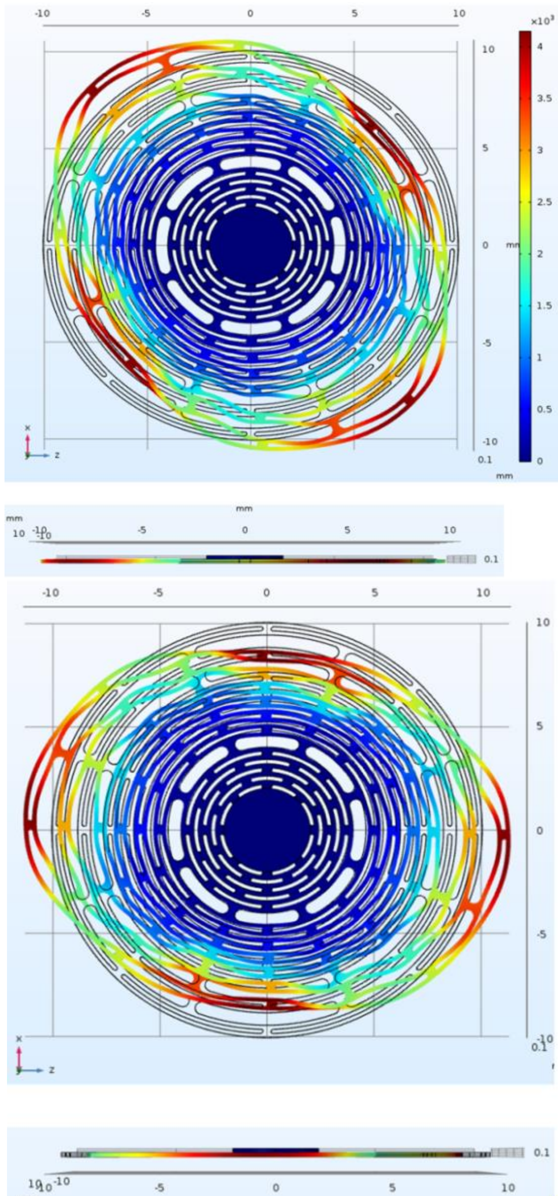

Figure 2: The (n=2) operating modes of a DRG resonator

From the loci presented in Figure 3 and Figure 4, several thickness regimes are identified within which coupling of modes can be avoided. These regimes are marked R_1 to R_5. From these regimes the following thicknesses are selected due to feasibility of manufacturing: 0.35mm, 0.52mm,0.75mm and 1.05mm. We take forward only these thicknesses for further Q-factor related investigation.

Thermo elastic damping

The mechanism of thermo elastic damping was first explained by Zener many decades ago [3]. He indicated that the phenomenon is induced by the irreversible heat dissipation during the coupling of heat transfer and strain rate in an oscillating system.

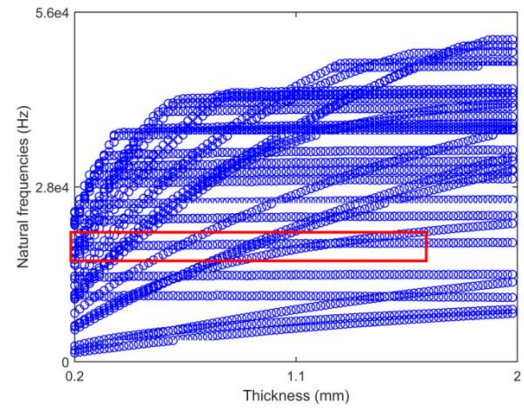

Figure 3: Loci of natural frequencies plotted against the thickness of the resonator

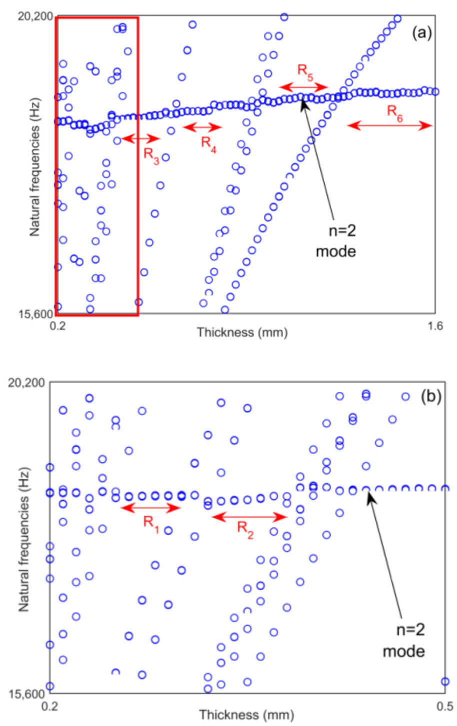

Figure 4: Magnified view of the loci of frequencies, falling within the region marked by the red rectangle of (a) Figure 3 and (b) Figure 4a.

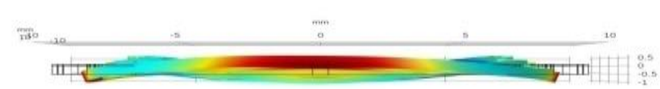

Figure 5: Effect of coupling on the n=2 mode shape.

When a beam is bent, one side of the beam is in tension and the other side is in compression. The side in compression gets slightly warmer and the side in tension gets slightly cooler due to the coupled nature of the thermal and mechanical domains. A temperature gradient is formed across the beam giving rise to heat flow for nonzero thermal conductivity and this heat flow is an irrecoverable energy loss that limits the quality factor of the beam.

The general heat conduction equation with heat generation is as follows.

$$k\nabla^2 T = \rho C_P \frac{\partial T}{\partial t} - \frac{E\alpha T_0}{1-2\vartheta}\nabla.\frac{\partial u}{\partial t}$$

k is the thermal conductivity, T is the temperature, ρ is the density, C_P is the specific heat capacity, E is the Young's modulus, α is the coefficient of thermal expansion, T_0 is the absolute equilibrium temperature, ϑ is the Poisson's ratio, u is the general displacement. The general equation of motion with additional thermal strain due to thermo elastic coupling is as follows:

$$\rho\frac{\partial^2 u}{\partial t^2} = \mu\nabla^2 u + (\mu+\lambda)\nabla(\nabla.u) - \frac{E\alpha}{1-2\vartheta}\nabla T$$

where μ and λ are the Lame's parameters. The above equations can be solved for complex eigen values, which will represent the dissipation. Here we use Comsol to estimate the Q-factors associated with resonators of the thicknesses selected in the previous section.

Thickness (mm)	Frequency of the n=2 mode (Hz)	Quality factor
0.35	16400	1.07E+08
0.52	16418	1.03E+08
0.75	16446	9.84E+07
1.05	16477	9.37E+07

Table 2: Frequencies and Quality factors for the n=2 mode at various thicknesses

It is observed that the Quality factor decreases with an increase in resonator thickness. Out of the four thickness studied in this section, the t=0.35mm case exhibited greatest Q-factor. Therefore for this thickness parameter, the effect of thin film coating is studied next.

DRG sensors are generally actuated electro-statically. The Quartz resonator is coated with a thin gold film so as to make it electrically conductive to facilitate actuation. However the gold coating reduces the Q-factor of the resonator. In this section we study the effect of a gold coating on Q-factor. We make use of the axisymmetric nature of the resonator section and therefore model one quadrant of the resonator and perform the study. The thickness of the coating was chosen to be 500nm. On analysis it was observed that the Q-factor reduced from 1.07e8 to 4e5.

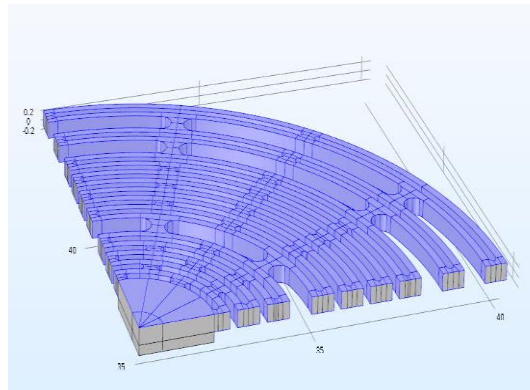

Figure 6: The model of a coated resonator.

Anchor loss

Yet another mechanism for Q-factor reduction in high Q systems is the energy losses at the attachment points. The cyclic forces and moments that resonator oscillation exerts on its anchors can excite shear and normal stress waves that propagate into the substrate; part of energy of these stress waves is absorbed by the anchors and the substrate and is dissipated. This phenomenon is referred to as anchor damping.

For beam resonators with the thickness much smaller than the transverse elastic wavelength the support loss due to moment is negligible compared to shear (as the radiation efficiency of the normal force is reduced since two halves of radiation source cancel each other). The shear force at a beam anchor point is evaluated in reference [4] based on the beam equation of resonant motion. This acts as the source of elastic waves in the support structure. Then, 2D elastic wave equation is used to generate closed form solution for anchor loss for clamped-free beam. The equation of motion of the in-plane flexural vibration of the beam resonator is as follows:

$$\frac{\partial^4 y}{\partial x^4} + \frac{\rho S}{EI}\frac{\partial^2 y}{\partial t^2} = 0$$

where I and S are the moment of inertia and cross-section area of beam respectively. The stored flexural vibration energy for n^{th} resonant mode is as follows:

$$W_n = \frac{1}{8}\rho S L \omega_n^2 U_n^2$$

where L is the length of the beam and U is the vibration amplitude. The vibrating shear force exerted by the resonator on its support is as follows:

$$\Gamma_n = EIU_n \left[\frac{\pi\beta_n}{L}\right]^3 \chi_n$$

where β_n is mode constant and χ_n is mode shape factor. This shear force excites elastic waves propagating into the support which carries energy away from the resonator and therefore reduces the Q-factor.

Anchor damping is expected to be approximately constant over temperature unlike thermo elastic damping. Often, this temperature independent characteristic of anchor damping is utilized to identify and determine the contribution of anchor damping in the measurements [5].
The Quality factor of a resonator considering the losses due to anchoring alone is investigated using Perfectly matched layers (PMLs). A PML is a finite domain that is attached to the outer boundary of a (finite element) model which contains the system of interest - in our case a resonator and part of the substrate (with possibly subsurface scatterers). The PML is a continuum domain with moduli devised in a fashion such that the mechanical impedance between the PML and the model is perfectly matched. This essentially eliminates spurious reflections from the artificial interface. The PML is finite in extent and thus has an outer boundary. The presence of an outer boundary requires the PML to damp the out-going waves before they reflect and pollute the computation [6].

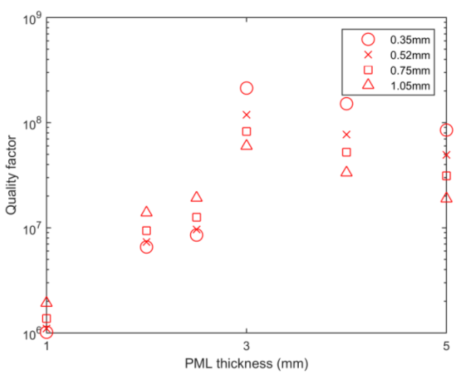

Figure 7: Variation of the estimated Quality factor with the thickness of the Perfectly matched layer for different resonator thicknesses.

In the work presented here, we obtain the Q-factors of resonators for the four thickness values selected in Section 4. For each of these four resonator thicknesses, the PML thicknesses are varied to find the optimum value (that which gives maximum Q-factor) and the associated Q-factors. Finally the thickness for which the obtained Q-factor is maximum, is selected. Figure 7 presents the variation of Quality factor as a function of the PML thickness for different resonator thicknesses. It is observed that for an analysis performed with low PML thickness (below 2.5 mm), thicker resonators exhibit higher Q-factors. However for PML thicknesses above 3mm, thinner resonators show higher Q-factors. The exact reason behind this trend needs further investigation. It is concluded that a resonator with thickness t=3mm shows maximum Q-factor and therefore is superior from an anchor loss perspective.

Summary

The aim of this work is to optimize the thickness of a disk resonator for a CVG by separating the functional mode from the other modes as well as maximizing Q-factor. Four thickness regimes where the frequency of the operating (n=2) mode is well separated from the frequencies of the other modes are identified. For each of these regimes, Q-factor estimation based on thermo elastic damping is performed. The effect of a conductive coating on the Q-factor is also investigated. It is found that the coating reduces the Q-factor 1000 times. The PML concept is Comsol is used for anchor loss simulation. Anchor loss estimation is done for different thicknesses. Finally taking into account the requirements of isolation of the operating mode's frequency and the maximization of Q-factor, an optimized geometry with a conductive coating layer is arrived.

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