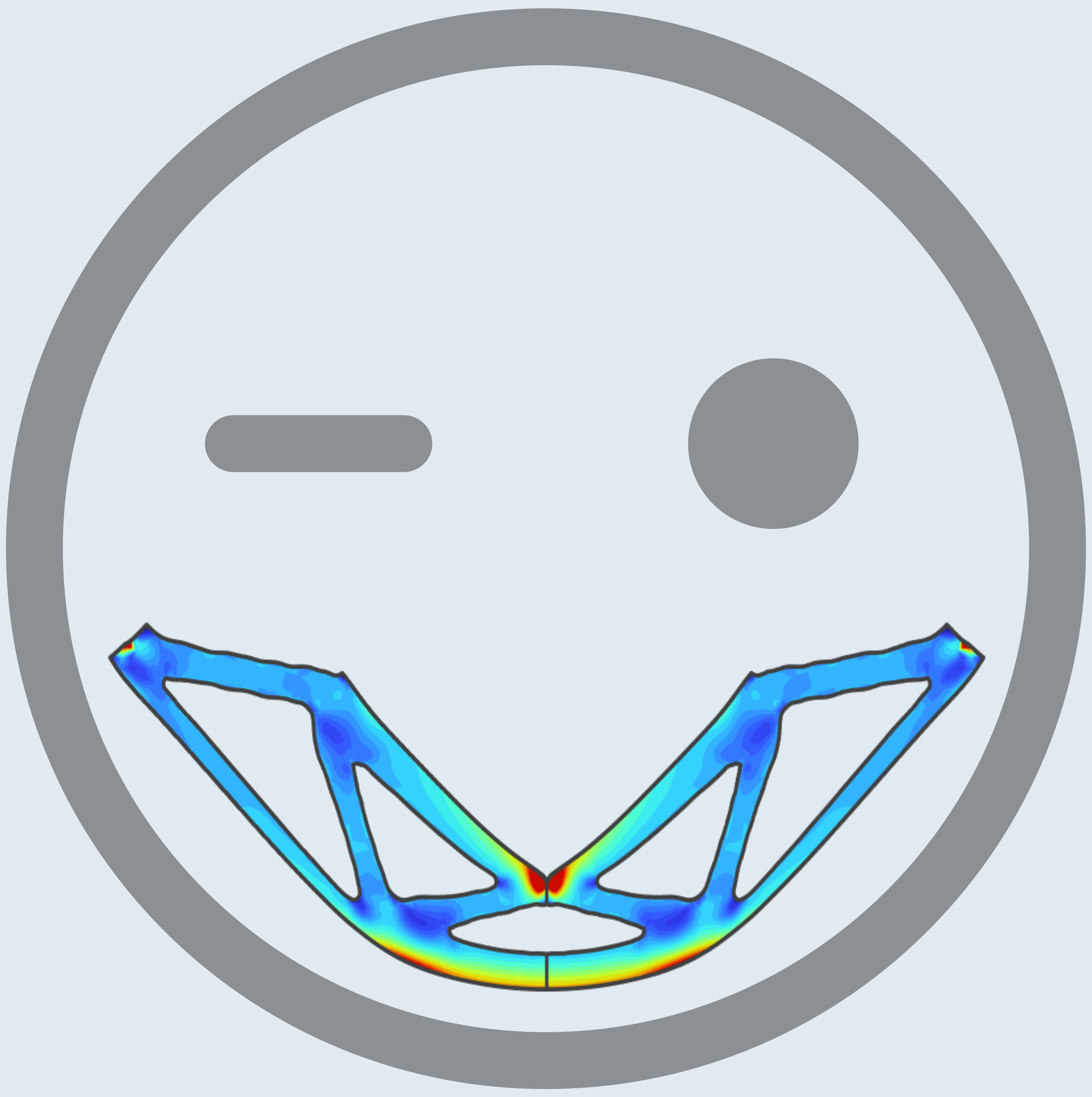

Out-of-the-box Objectives

Exploring probabilistic failure functionals as novel objectives in COMSOL® structural optimization.

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Abstract

In structural optimization, the norm is to minimize either peak stress or compliance under load. However, probabilistic quantities, such as the probability of failure, offer a fresh and powerful alternative. These quantities are inherently dimensionless, confined to [0,1], and provide a comprehensive assessment of the overall design quality by considering the stress distribution across the entire structure. Here we explore using the Weibull integral to quantify the probability of failure for brittle materials, such as ceramics or cast iron, as well as failure due to fatigue under cyclic loading. By addressing the question, **"How can I minimize the total probability that my design will fail?"** we demonstrate how these probabilistic objectives can be integrated into COMSOL optimization methods, including topology and shape optimization. This approach enhances design reliability and enables more efficient, resilient structures tailored to specific load scenarios.

FIGURE 1. Probability of failure: TOO LARGE! Collapsed Carola-Bridge in Dresden, Germany.

Methodology

- The **Weibull Integral** gives the probability of failure P_f as a functional of stress distributions $\sigma(\vec{x})$ across the structure:

$$P_\mathrm{f} = 1 - \exp\left[-\frac{1}{V}\int_V \left(\frac{\sigma(\vec{x})}{\sigma_0(N)}\right)^m \mathrm{d}V\right]$$

- Implementing P_f in COMSOL can be realized with a *nonlocal coupling integral* over all domains of a structural component.
- Can be used as **objective to be minimized** in any **topology or shape optimization study**.
- For fatigue analysis, simply plug in appropriate S-N curve for the reference stress $\sigma_0(N)$.

Results

- We applied the Weibull formalism to the COMSOL MBB beam example, demonstrating how it can be used to further refine the beam's shape for optimal durability under cyclic loads or for brittle materials like concrete.
- The Weibull Integral offers a new perspective on how different regions and stresses contribute to a **global probability of failure and how to effectively minimize it.**
- This approach is particularly suited for structural components made from multiple materials, each with unique Weibull and S-N relations, as well as for combining multiple load cases.

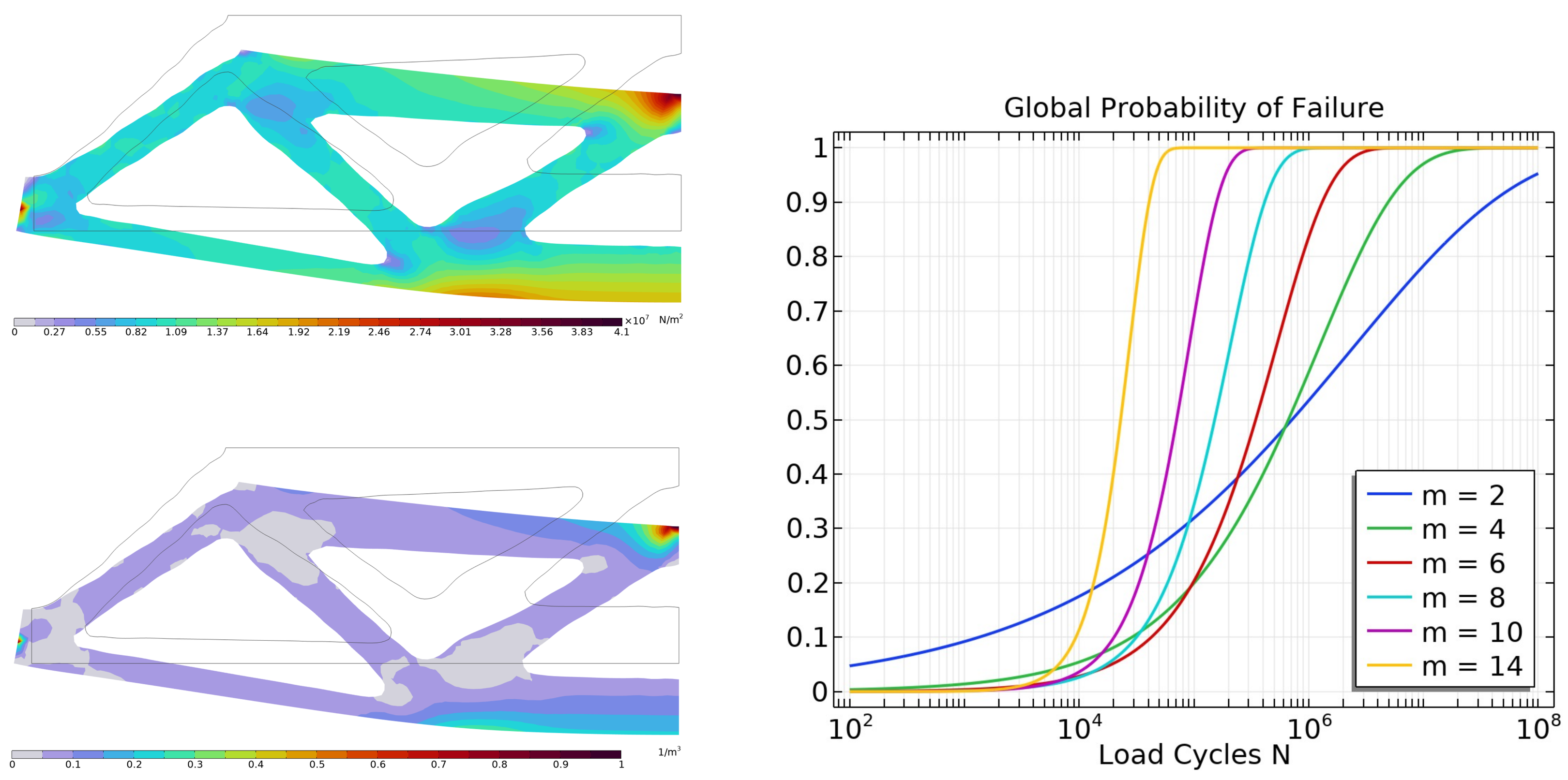

FIGURE 2. **Left:** Mises stress (upper) and corresponding Weibull Density (lower) for the familiar MBB beam under 3-point loading. **Right:** Global probability of failure for various Weibull Moduli *m*, assuming cyclic loading and a Basquin power-law for the S-N relation.

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