Floating Potential with Integral Constraints

This example shows how to treat integral constraints on the flux on a boundary with a constant but unknown value of the dependent variable. Depending on the physics studied, the following phenomena can be described:

- In electrostatics:

  The modeling of a floating electrode with a fixed total charge.
- In conductive media:

  The modeling a floating electrode with a fixed total current.
- In diffusion:

  The modeling of a reaction on a boundary with a fixed total reaction rate and a fixed but unknown concentration.

For simplicity, we choose to model an electrostatics problem with one floating electrode carrying a fixed charge. The technique is analogue for the other problems.

Problem Definition

The geometry consists of a quadratic dielectric domain. Inside the domain we have a curve representing an electrode with a fixed charge, Q_{ref}.

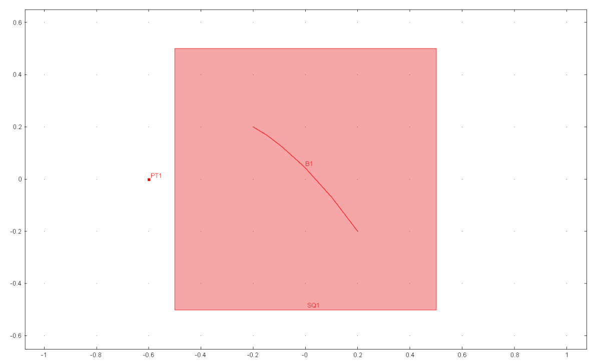

Figure 0-1: Geometry.

DOMAIN EQUATIONS

The model equation is the Laplace equation for electrostatics:

$$-\nabla \cdot (\varepsilon \nabla V) = 0$$
$$\mathbf{E} = -\nabla V$$

where ε denotes the permittivity, V the potential and $\mathbf{E}$ the electric field.

BOUNDARY CONDITIONS

The upper boundary has the potential V=1 and the lower boundary is connected to ground, V=0. The left and right boundaries are insulated i.e. $\mathbf{n}\cdot\mathbf{E}=0$. The floating boundary has an unknown potential V_0 which depends on Q_{ref}.

Remarks About the FEMLAB Implementation

By referring to the unknown potential V_0, we introduced an extra degree of freedom in the problem. Thus, an extra equation must be added in order to obtain a well-posed problem. The extra equation is introduced in a single point as a weak point application mode. A dedicated point outside of the main geometry is added for this purpose.

The weak point application mode must be supplied with a measure of the total charge on the electrode. Therefore, we add a weak boundary constraint application mode to the electrode boundary. The exact measure of the charge density on the electrode is given by $-lm$ where lm denotes the dependent variable for the weak point application mode, the so-called Lagrange multiplier. A coupling variable Q_{tot}, is introduced which integrates the negative of the weak boundary constraint variable, $-lm$, over the electrode and sends the result to the weak point mode. In the weak point mode, we set the difference Q_{ref}-Q_{tot} to zero, where Q_{ref} is a model constant representing the desired total charge on the electrode. Finally, the dependent variable of the weak point mode is coupled via another variable back to the electrode as the unknown potential V_0. In this way, we have defined the system of equations required to find the constant but unknown potential of the electrode.

Results

Figure 0-2 below shows that the potential is kept constant on the electrode boundary as desired.

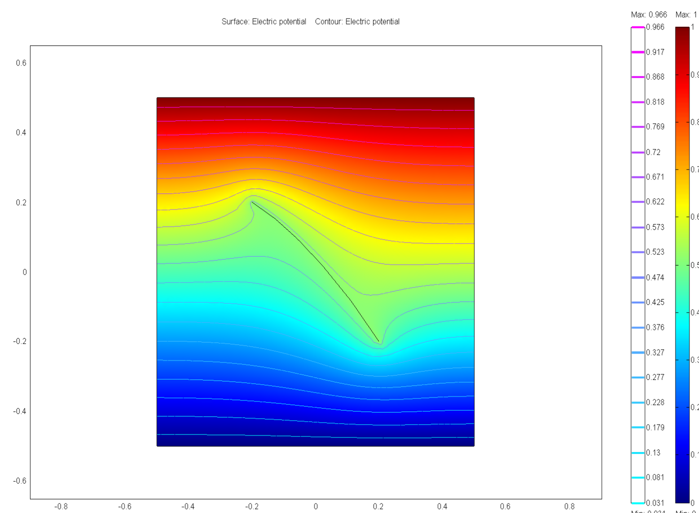

Figure 0-2: Potential distribution.

The current density varies substantially along the electrode and the edge effects are clearly shown in Figure 0-3. The plate works as a bipolar electrode where the current density is positive in some parts and negative in other.

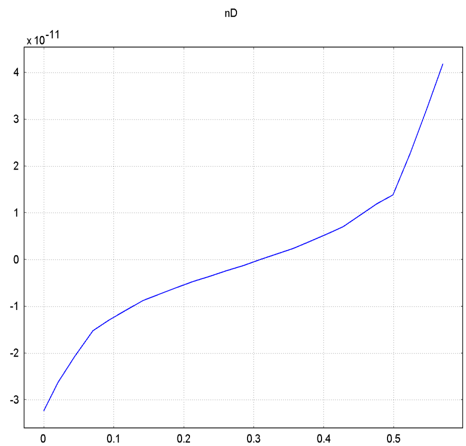

Figure 0-3: Current density along the electrode

The example shows the possibility of postulating a constant potential without knowing the value apriori. The weak boundary constraint feature makes it possible.

Modeling Using the Graphical Users Interface

1 Open FEMLAB.
2 Make sure the **Space dimension** is **2D**.
3 Click the **Multiphysics** button.
4 Select **Electrostatics** from the **Electromagnetics** folder and click **Add**.
5 Click the **Application Mode Properties** button and change **Weak constraints** to **non-ideal** and click **OK**.
6 Select **Weak Form, Point** from the **PDE Modes** folder
7 Change the **Dependent variable** to `V0` and click **Add**.
8 Click **OK**.

OPTIONS AND SETTINGS

1 Select **Constants** from the **Options** menu.
2 Enter the constant `Qref` with the value `0`.

GEOMETRY MODELING

1 Draw a square with width 1 centered in `(0,0)`.
2 Dubble click on the text **SOLID** in the bottom of the window.
3 Draw a curve inside the square along which the potential should be constant.
4 Draw a point left of the square.

PHYSICS SETTINGS

Boundary Settings

1 Select **Electrostatic** from the **Multiphysics** menu.
2 Open the **Boundary Settings** dialog box from the **Physics** menu.
3 Check the **Interior boundaries** check box and enter the following boundary conditions:

TABLE 0-1:

BOUNDARY	1, 4	2	3	5
Type	Electric insulation	Ground	Electric potential	Electric potential
V			1	V2

4 On the **Weak Constr.** page clear the **Use weak constriants** checkbox for boundary **1-4**.
5 Check this checkbox for boundary **5** and set **Predefined elements** to **Lagrange-Quadratic.**

Subdomain Settings

1 Open the **Subdomain Settings** dialog box.
2 Make sure the relative permittivity is `1` and the space charge density is `0`.

Point Setting

1 Select **Weak Form, Point** from the **Multiphysics** menu.
2 Open the **Point Settings** dialog box fromt the **Pysics** Menu.
3 On the **Weak** page select point **1** and enter the following expression in the **Weak** field: `-V0_test*(Qtot-Qref)`
4 Select points **2-7** and clear the **Active in this domain** checkbox.

COUPLING VARIABLES

1 Select **Integration Coupling Variables** and then **Boundary Variables** from the **Options** menu to open the **Boundary Integration Variables** dialog box.

2 Select boundary 5 and define the following variable:

TABLE 0-2:

SOURCE	
Name	`Qtot`
Expression	`-lm1`
Integration order	4

3 Clear the **Global destination** checkbox.

4 On the **Destination** page set the **Level** to **Point** and check point number 1.

5 Click **OK**.

6 Open the **Point Integration Variables** dialog box.

7 Select point number 1 and enter a variable with **Name** `V2` and **Expression** `V0`.

8 Clear the **Global destination** checkbox.

9 On the **Desitination** page set the **Level** to **Boundary** and check boundary 5.

MESH GENERATION

Initialize the mesh.

COMPUTING THE SOLUTION

Click the **Solve** button to solve the model.

POSTPROCESSING AND VISUALIZATION

1 In **Plot Parameters** check the checkboxes for **Surface** and **Contour** plot types.

2 On the **Surface** page select **Electric Potential** as **Surface data**.

3 On the **Contour** page select the **Electric Potential** as **Contour data**.

4 Click **OK**.