Scattering characteristics of simplified cylindrical invisibility cloaks

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Abstract: The previously reported simplified cylindrical linear cloak is improved so that the cloak's outer surface is impedance-matched to free space. The scattering characteristics of the improved linear cloak is compared to the previous counterpart as well as the recently proposed simplified quadratic cloak derived from quadratic coordinate transformation. Significant improvement in invisibility performance is noticed for the improved linear cloak with respect to the previously proposed linear one. The improved linear cloak and the simplified quadratic cloak have comparable invisibility performances, except that the latter however has to fulfill a minimum shell thickness requirement (i.e. outer radius must be larger than twice of inner radius). When a thin cloak shell is desired, the improved linear cloak is much more superior than the other two versions of simplified cloaks.

OCIS codes: (290.5839) Scattering, invisibility; (260.0260) Physical optics; (999.9999) Invisibility cloak.

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1. Introduction

Some recent works have indicated that it might be possible to realize passive invisibility cloaking devices [1, 2, 3]. Based on coordinate transformation of Maxwell equations, Pendry et al.

#88913 - $15.00 USD Received 22 Oct 2007; revised 4 Dec 2007; accepted 5 Dec 2007; published 13 Dec 2007

have suggested that a perfect invisibility cloak can be constructed for perfect hiding of arbitrary objects from electromagnetic (EM) illumination [1]. The direct coordinate transformation methodology tears up the original coordinate space (see Fig. 1), which leads to a cylindrical cloak with extreme material parameters at the cloak's inner boundary. In view of this difficulty, a cylindrical invisibility cloak as suggested by the coordinate transformation, though proved to be asymptotically perfect [4], has to be realized in an approximate manner. The simplest method to avoid the extreme material parameters is to peel away a thin layer from an ideal cloak's inner surface. Such a cloak can be made arbitrarily close to an ideal one [4]. Another proposed method is to modify the inner surface of the cloak [5]. Under these two schemes, the cloak still needs to have three gradient material parameters per EM polarization (TE or TM). In [6, 7], the authors proposed that, for a single-polarization in-plane propagation, the cloak can be simplified (based on the ideal cloak derived from linear coordinate transformation) such that it has only one gradient material parameter. In addition, the requirement on infinite material parameter is lifted. This greatly eases the challenge in constructing a cloak by using man-made metamaterials. An almost identical simplification procedure was also adopted in [8] for a non-magnetic cloak, and later for a cloak based on quadratic coordinate transformation [9].

With an arbitrary scatterer placed within a simplified linear cloak, EM field scattered by the overall structure may be smaller than that scattered by the bare scatterer. But, the performance of such a simplified cloak is still found to deviate much from an ideal invisibility cloak (which should have zero scattering) [10]. We will show in Section 2 that the mathmatical backbone of the simplification procedure proposed in [6] is not stringent. Despite lack of rigorousness, a cloak resulted from such a simplification, however, inherits some properties of an ideal cloak [6, 7, 10], especially in its tendency to bend light around the inner shell. In this paper, we show that it is possible to improve the simplified linear cloak so that its induced scattering can be significantly lower. We start by noticing that the simplified material parameters presented in [6] is not the only solution according to the simplification procedure. In fact, a better simplified cloak with improved invisibility performance can be derived. The improvement is obtained by matching the impedances of the cloak and the outside space at the cloak's outer surface. We will also compare the improved linear cloak with the simplified cylindrical cloak derived from a quadratic coordinate transformation, which similarly can perfectly match with the outer space in impedance at its outer surface [9]. It is found that the improved linear cloak has the superiority of being consistently near-invisible even when the cloak has a very thin wall. In addition, from the engineering point of view, the material requirement for realizing an improved linear cloak is also lower.

Before we proceed, we should recount the two necessary conditions for claiming a perfect invisibility cloak. Firstly, the device should have a spacial region that is in complete EM isolation with the outer space. Secondly, the device should cause no perturbation to any incident EM field. The cloaks in simplified forms will most likely disqualify for both requirements. That is, they would allow a certain percent of foreign field to penetrate into the cylindrical shell, apart from causing scatterings. A quick and reliable fix to the first problem is to introduce a pefect electric/magnetic conductor (PEC/PMC) lining on the inner surface of the cloak shell, as has been mentioned in some previous works [6, 8]. Now the only problem left is how to minimize the cloak's scattering by using various material simplification schemes, based on the ideal coordinate-transformed cloaks. From a practical point of view, it is more appropriate to consider the PEC/PMC lining as one part of the cloak body, rather than an object that we are trying to hide. In this paper, we concentrate on simplified cloaks *with* PEC/PMC linings. The scattering characteristics of simplified cloaks without PEC/PMC linings depend heavily on objects that the cloaks enclose. Their performance will only be briefly remarked.

This paper is organized as follows. The theoretical equations that are necessary for under-

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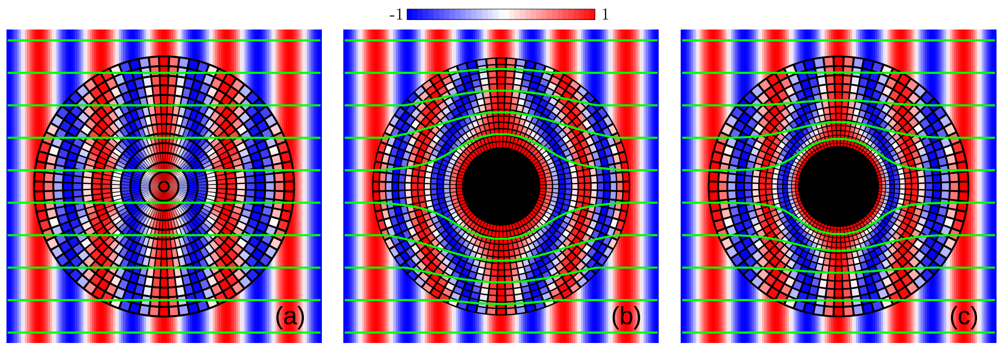

Fig. 1. Illustrations of interactions between a plane wave and perfect cylindrical cloaks resulted from coordinate transformations. (a) Original EM space; (b) Ideal linear cloak; (c) Ideal quadratic cloak. The colormap is for E_z field. Green lines are for energy flow (poynting vector). a=1, b=3, $\lambda = 1.5$ (a.u.). Domain size is at 7.5×7.5. For the quadratic cloak, $p = a/b^2$. (Read the text for parameter descriptions.) Invariant coordinate lines are imposed to show how the EM fields are compressed.

standing simplified cloaks based on both linear and quadratic coordinate transformations are presented in Section 2. The inherent scattering properties of the improved linear cloak and the simplified quadratic cloak are studied in Section 3. A comparison of the performances of simplified cloaks subject to a plane wave incidence is presented in Section 4. A conclusion follows in Section 5.

2. Theoretical background

2.1. Simplified linear cloaks

In this paper, we restrict our analysis on normal incidence. That is, the wave vector is perpendicular to the cloak cylinder axis. The EM wave is assumed to have TE polarization (i.e. electric field only exists in z direction). The TM polarization case can be derived by making $E \to -H$, $\varepsilon \to \mu$, $\mu \to \varepsilon$, and PEC→PMC substitutions. Domains inside and outside the cloak are both assumed as air.

The ideal cloak is supposed to compress all fields within a cylindrical air region $r < b$ into the cylindrical annular region $a < r < b$ (Fig. 1). This can be achieved by a cylindrical coordinate transformation from the virtual EM space (r', θ', z') to physical space (r, θ, z) [1, 11], where $\theta = \theta'$ and $z = z'$. In general r is a function of r' and with the conditions $r|_{r'=o} = a$ and $r|_{r'=b} = b$. The transformation, in the simplest linear case, can be written as

$$r = \frac{b-a}{b} r' + a. \tag{1}$$

According to invariance of the Maxwell equations, the corresponding material parameters can be obtained, as shown in the left column of Table 1. This set of parameters are plotted in Fig. 2(a) (in black curves) for a sample cloak. Notice the μ_θ (and ε_θ) is infinite at $r = a$. The "perturbation" caused by such an ideal cloak, as it is illuminated by a plane wave, is illustrated in Fig. 1(b). To circumvent the fabrication difficulty, the simplified parameters were used [6, 8], which are given in the middle column of Table 1. This set of parameters are shown by the green curves in Fig. 2(a). To understand the motivation behind such a simplification, we have to start from the derivation of the wave equation.

The cloak, together with its inner and outer air domains, form a three-layered cylindrical EM system. We refer to the layers from inside to outside as layer 1, 2 and 3, respectively. Within

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Table 1. Material parameters for linear cloaks

Ideal	Simplified [6]	Simplified (current work)
$\varepsilon_r = \mu_r = \frac{r-a}{r}$	$\varepsilon_r = \mu_r = \left(\frac{r-a}{r}\right)^2$	$\varepsilon_r = \mu_r = \left(\frac{r-a}{r}\right)^2 \frac{b}{b-a}$
$\varepsilon_\theta = \mu_\theta = \frac{r}{r-a}$	$\varepsilon_\theta = \mu_\theta = 1$	$\varepsilon_\theta = \mu_\theta = \frac{b}{b-a}$
$\varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r}$	$\varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)^2$	$\varepsilon_z = \mu_z = \frac{b}{b-a}$

the cloak medium, the general wave equation that governs the E_z field can be written as

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(\frac{r}{\mu_\theta}\frac{\partial E_z}{\partial r}\right)\right] + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\mu_r}\frac{\partial E_z}{\partial \theta}\right) + k_0^2 \varepsilon_z E_z = 0, \tag{2}$$

where k_0 is the free-space wave number. The cloak's coordinate is chosen as the global coordinate. The time dependence $\exp(i\omega t)$ has been used for deriving Eq. 2. By employing the variable separation $E_z = \Psi(r)\Theta(\theta)$, we can decompose Eq. 2 into two separate equations. One of them leads to $\Theta = \exp(im\theta)$, where m is an integer. The other r-dependent equation is

$$\frac{d}{dr}\left(\frac{r}{\mu_\theta}\frac{d\Psi}{dr}\right) + k_0^2 r \varepsilon_z \Psi - m^2 \frac{1}{r\mu_r}\Psi = 0. \tag{3}$$

The simplification procedure as described in [6, 7] argues that *if μ_θ is r-independent*, Eq. 3 can be further written as

$$\frac{1}{\mu_\theta \varepsilon_z}\frac{d}{dr}\left(r\frac{d\Psi}{dr}\right) + k_0^2 r \Psi - m^2 \frac{1}{r\mu_r \varepsilon_z}\Psi = 0. \tag{4}$$

Unfortunately, the assumption of a constant μ_θ is invalid in the very first place. As a result, the wave equation 4 has deviated from the original wave equation 3. The simplication procedure, now based on Eq. 4, claims that the wave equation will not be changed as long as the functionals $\mu_\theta \varepsilon_z$ and $\mu_r \varepsilon_z$ are kept invariant, with respect to their values derived from the ideal parameters. Therefore one has the freedom in choosing the metamaterial parameters. For the particular experiment in [6], material parameters given in the middle cloumn of Table 1 are used.

An important consequence of the simplification is that, besides a change of the wave equation within the cloak medium, the outer surface of the cloak is not perfectly matched with free space. Originally the perfectly matched layer (PML) condition, i.e. $\varepsilon_z = \mu_\theta = 1/\mu_r$, is valid at $r = b$. Now the impedence at $r = b$ instead becomes $Z = \sqrt{\mu_\theta/\varepsilon_z} = (b-a)/b$, rather than 1. The cloak hence inevitably scatters any incoming EM field. The overall scattering coefficients, especially for the zeroth-order cylindrical wave component, can still be significant even when the cloak's outer radius is kept several times as large as its inner radius [10]. Despite the shortcomings, the scattering characteristics of such a simplified cloak is noticed to be distinct as compared to a conventional annular cylinder made of an isotropic material or simply a PEC cylinder. An incident EM wave has the tendency to flow around the cloak's inner boundary, much like (but not exactly) the way it behaves in an ideal cylindrical cloak.

However, one can notice that, to keep the functionals $\mu_\theta \varepsilon_z$ and $\mu_r \varepsilon_z$ invariant with respect to the ideal values, more than one solution exist. In the right column of Table 1, we show another set of material parameters satisfying the requirement. For the new parameter set, at $r = b$ we have $\varepsilon_z = \mu_\theta = 1/\mu_r = b/(b-a)$, i.e. the PML condition. With the impedance-matched outer surface, we expect that a cloak with the new set of parameters would induce smaller scattering. The improved parameters are plotted in Fig. 2(a) in red curves. Notice the unchanged material values at $r = b$ as comapred to those for the ideal cloak (black curves).

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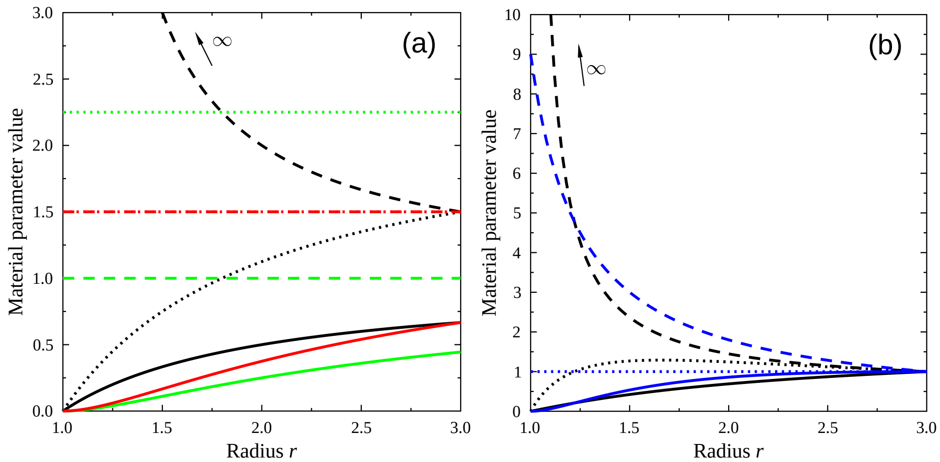

Fig. 2. (a) Material parameters for an ideal linear cloak (black curves), a simplified linear cloak (green curves), and an improved linear cloak (red curves). (b) Material parameters for an ideal quadratic cloak (black curves), and an simplified quadratic cloak (blue curves). Solid line: μ_r; dashed line: μ_θ; dotted line: ε_z. All cloaks have $a = 1$ and $b = 3$ (a.u.).

We go a bit further by substituting parameters in the middle or the right column of Table 1 into the wave equation 4. We obtain

$$(r-a)^2 \frac{d^2\Psi}{dr^2} + \frac{(r-a)^2}{r}\frac{d\Psi}{dr} + \left[(r-a)^2\left(\frac{b}{b-a}\right)^2 k_0^2 - m^2\right]\Psi = 0. \tag{5}$$

When $m = 0$, Eq. 5 can be further simplified to

$$r^2 \frac{d^2\Psi}{dr^2} + r\frac{d\Psi}{dr} + r^2 k_0^2 \left(\frac{b}{b-a}\right)^2 \Psi = 0. \tag{6}$$

This is simply the zeroth-order Bessel differential equation. Equation 6 suggests that an incoming zeroth-order cylindrical wave would effectively see the simplified cloak as a homogeneous isotropic medium whose effective refractive index is $n_{\text{eff}} = \frac{b}{b-a}$ [10].

2.2. *Simplified quadratic cloak*

In a very recent paper, the quadratic coordinate transformation was proposed for the cloak design [9]. It is found that the simplified version of the quadratic cloak can also match with the outer free space in impedance. The quadrtic coordinate transformation takes the form of

$$r = \left[\frac{b-a}{b} + p(r'-b)\right] r' + a, \tag{7}$$

where p can be arbitrary as long as $|p| < (b-a)/b^2$ is valid for keeping the spacial mapping monotonic. The ideal material parameters according to such a coordinate transformation are shown in the left column of Table 2. Since p is free to change here, the material parameters can be varied. Interestingly at $p = a/b^2$, we have $\frac{dr}{dr'}\big|_{r=b} = 1$. Under this condition, the material parameters are all valued at 1 at $r = b$. Electromagnetically, the resulted cloak does not even have an interface at $r = b$. The anisotropic ratio of the cloak material then grows at radial positions closer to the cloak's inner surface. Notice this particular choice of p value, together

#88913 - $15.00 USD Received 22 Oct 2007; revised 4 Dec 2007; accepted 5 Dec 2007; published 13 Dec 2007

with the monotonicity condition, impose a constraint on the *ideal* cloak's wall thickness, which is $b > 2a$. The material parameters for a sample ideal cloak are plotted in Fig. 2(b) (black curves). Again divergent μ_θ is observed at $r = a$. Figure 1(c) shows such a quadratic cloak in interfaction with a plane wave. In [9], the authors again start from the modified wave equation 4 and propose that the cloak can be simplified by keeping functionals $\mu_\theta \varepsilon_z$ and $\mu_r \varepsilon_z$ invariant. The simplified cloak has the material parameters as given in the right column of Table 2. The simplified material parameters are plotted in Fig. 2(b) (also with $p = a/b^2$). Notice that the "interfaceless" property at $r = b$ is inherited by the simplified cloak.

Table 2. Material parameters for quadratic cloaks

Ideal	Simplified [9]
$\varepsilon_r = \mu_r = \frac{r'}{r}\left[p(2r'-b) + \frac{b-a}{b}\right]$	$\varepsilon_r = \mu_r = \left(\frac{r'}{r}\right)^2$
$\varepsilon_\theta = \mu_\theta = \frac{1}{\varepsilon_r}$	$\varepsilon_\theta = \mu_\theta = \frac{1}{\left[p(2r'-b)+\frac{b-a}{b}\right]^2}$
$\varepsilon_z = \mu_z = \frac{r'}{r}\frac{1}{p(2r'-b)+\frac{b-a}{b}}$	$\varepsilon_z = \mu_z = 1$

Strictly speaking, the monotonicity condition, a requirement by the coordinate transformation, is only valid for the ideal cloak. The simplified cloak is not directly obtained from the coordinate transformation. Hence it may be questionable if the monotonicity, and hence the minimum thickness constraint ($b > 2a$), are still applicable for the simplified cloak. We will show later the significance of the wall thickness to the performance of simplified quadratic cloaks. Another issue worth noting is that, by starting from the modified wave equation 4, it has already been assumed that μ_θ is a constant. However, the simplification process ends with a μ_θ parameter that is dependent on radius r. Nevertheless, such a cloak with a PEC lining was shown to be partially invisible [9]. The simplified quadratic cloak will be compared to the simplified linear cloaks in Sections 3 and 4.

2.3. Analysis procedure

Due to the fact that the wave equation inside a cylindrical cloak is separable, the scattering characteristics of a cloak can be examined in each cylindrical order. The obtained scattering coefficients in different orders are source independent. That is, they are inherent properties of a cloak.

The calculation of scattering coefficients in each cylindrical order follows the procedure as described in [10]. In short, the radial dependences of fields in three layers of the EM system, for a fixed m, can be written as

$$E_z^1 = \sum_m \mathscr{A}_m^1 J_m(k_0 r)\exp(im\theta), \quad (8)$$

$$E_z^2 = \sum_m \{\mathscr{A}_m^2 Q_m + \mathscr{B}_m^2 R_m\}\exp(im\theta), \quad (9)$$

$$E_z^3 = \sum_m \{\mathscr{A}_m^3 J_m(k_0 r) + \mathscr{B}_m^3 H_m^{(2)}(k_0 r)\}\exp(im\theta). \quad (10)$$

In Eqs. 8-10, the transcendental functions Q_m and R_m are solutions to the cylindrical wave equation in the cloak medium, which depend on r; J_m and $H_m^{(2)}$ are Bessel function and Hankel function respectively; $\mathscr{A}_m^i$ and $\mathscr{B}_m^i$ are coefficients. The J_m and $H_m^{(2)}$ terms in the 3rd layer are physically in correspondence to the incident and scattered waves, respectively. Hence, the scattering problem becomes to solve for, most importantly, $\mathscr{A}_m^1$ (transmitted field) and $\mathscr{B}_m^3$ (scattered field) subject to a given incidence $\mathscr{A}_m^3$. The scattering and transmission coefficients

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in each cylindrical order are defined as $s_m = \left|\mathscr{B}_m^3/\mathscr{A}_m^3\right|$ and $t_m = \left|\mathscr{A}_m^1/\mathscr{A}_m^3\right|$, respectively. When $m = 0$, the scattering and transmission coefficients can be deduced analytically as Q_m and R_m are merely Bessel and Hankel functions respectively.

The scattering problem is tackled by using the finite-element method (FEM) with the commercial COMSOL Mutiphysics package. The incoming cylindrical wave is generated by a circular current source surrounding the cloak. The derived fields outside and inside the cloak are used to derive coefficients $\mathscr{A}_m^3$, $\mathscr{B}_m^3$, and $\mathscr{A}_m^1$ through a fitting procedure. Of course, if the cloak has a PEC lining, $\mathscr{A}^1$ and hence t_m are simplify zero. s_m and t_m can be calculated in turn. The validity of this procedure will be confirmed by comparing the numerical results with the analytical results (for simplified linear cloaks and when $m = 0$).

3. Inherent scattering properties

The invisibility performance of an ideal cylindrical cloak does not vary with respect to the cloak scale or wavelength. However, cloaks are imperfect once they are simplified. They can become cavities and have their own resonance conditions. Their invisibility performances in turn are influenced by the cloak scale and the wavelength. Three key parameters, namely the cloak shell's inner radius a, outer radius b, and the free-space wavelength λ (or frequency f) are affecting the cloak's performance. Detailed effects of these three parameters will be presented. Recall that the Maxwell equations are scale-invariant. An increase (decrease) in λ is equivalent to a decrease (increase) of a and b together in proportion. Focus will be given to simplified cloaks with a PEC lining.

3.1. Simplified cloaks with a PEC lining

Three types of cylindrical cloaks, including the previously proposed simplified linear cloak [6, 7], improved simplified linear cloak, and the simplified quadratic cloak are presented in this sub-section. All cloaks consist of a PEC lining. When the effect of b is analyzed, we fix $a = 0.1$m and $f = 2$GHz (or $\lambda = 0.15$m). When the effect of a is analyzed, we fix $b = 0.6$m and frequency is unchanged. When the the effect of λ is analyzed, we fix $a = 0.2$m and $b = 0.6$m. The calculated scattering coefficients for the three types of simplified cloaks are summarized in Fig. 3. Panels in each row in Fig. 3 are for a particular type of cloak, while panels in each column are for the effect of a particular parameter. Only the scattering coefficients for the first four cylindrical orders are presented. In general, for all cloaks the scattering is smaller when the order number m is larger.

From Fig. 3, it is quite evident that the originally proposed simplified linear cloak [Fig. 3(a1)-(a3)] induces relatively larger scattering in all cylindrical orders compared to the other two cloaks. The scattering in each order vary greatly as a, b or λ changes, suggesting the relatively strong cavity effect of the cloak. In a dramatic contrast, the scatterings caused by the improved linear cloak [Fig. 3(b1)-(b3)] have only very slight dependence on either a, b or λ. The geometry-insensitive scattering characteristics should be attributed to the perfect impedance matching at the cloak's outer surface. The EM system now does not have a closed region bounded by reflective interfaces. Therefore a cavity can't be formed. For the simplified quadratic cloak [Fig. 3(c1)-(c3)], the scattering characteristics is quite similar to that of the improved linear cloak when its geometry fulfills the $b > 2a$ condition. The relatively small scatterings are again due to the matched outer surface with the exterior space. With a relatively thin cloak wall ($b < 2a$), incoming cylindrical waves in general experience heavy scatterings. This result confirms the applicability of the minimum thickness requirement, i.e. $b > 2a$, for the simplified quadratic cloak.

One interesting observation from Fig. 3 is that, for all three types of cloaks, the scattering coefficient in each cylindrical order tends to converge to some specific value when $b \gg a$. For

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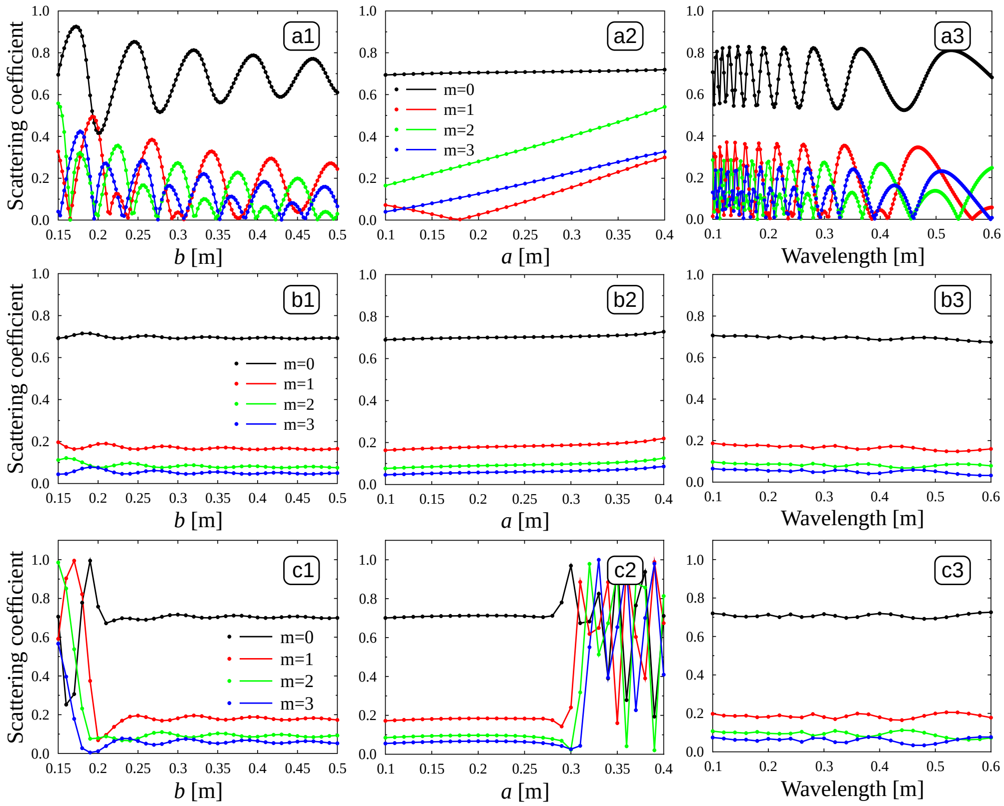

Fig. 3. The scattering coefficients in each cylindrical order as a function of either b, a or λ. (a1)-(a3) Previously proposed simplified linear cloak; (b1)-(b3) Improved simplified linear cloak; (c1)-(c3) Simplified quadratic cloak. In each row, the left panel shows the effect of b as $a = 0.1$m and $\lambda = 0.15$m; the middle panel shows the effect of a as $b = 0.6$m and $\lambda = 0.15$m; the right panel shows the effect of λ as $a = 0.2$m and $b = 0.6$m.

example, the zero$^{\text{th}}$-order scattering coefficient s_0 is apprroaching to $\sim$0.7, and s_1 is approaching to $\sim$0.16, etc. As the geometrical ranges of cloaks analyzed in this study are considered quite wide, the values suggests that the performances of the simplified cloaks may have an upper limit. For the improved linear cloak and the simplified quadratic cloak, scatterings are dominated by the zero$^{\text{th}}$-order component. For the originally proposed simplified linear cloak, relatively heavier scattering exists even when $b \gg a$. In addition, its scattered field is likely to comprise a significant portion of high-order cylindrical waves.

3.2. Remark on simplified cloaks without PEC lining

In this sub-section, we briefly look into the scattering property of simplified cloaks without a PEC lining. Most importantly, once the PEC lining is absent, field is able to penetrate into the cloak. As a consequence, the air column bounded by $r = a$ becomes a cavity, which leads to geometry-dependent scattering properties for all cloaks. In Fig. 4, we show the calculated results for the improved linear cloak. The scattered field is observed to be affected strongly by resonance/anti-resonance of the cavity, especially in Fig. 4(b) and (c). This structure allows an analytical derivation of the zero$^{\text{th}}$-order scattering coefficient [10]. Excellent agreement between the analytical and numerical results are noticed in Fig. 4, which confirms the validity of the deployed numerical procedure. It should be noticed that the curves in Fig. 4 will be different if an object is present inside the cloak, due to a change of the cavity resonance condition.

The major difference noticed by comparing Fig. 4(a)-(c) and Fig. 3(b1)-(b3) is that the curves for the zeroth-order scattering coefficient have been changed greatly. Almost no change is noticed for the high-order curves. From our numerical calculations, for $m > 0$ and when no PEC is present, the E_z field has almost decayed to zero when the wave approaches to the cloak's inner surface. This explains why the high-order curves are not affected after a PEC lining is imposed. In contrast, the zeroth-order cylindrical wave sees the cloak as an isotropic homogeneous material, so it is now free to go into and out of the cloak as governed by the cloak's resonance condition. Hence, the scattered field is a strong function of the cloak geometry, especially the parameter a.

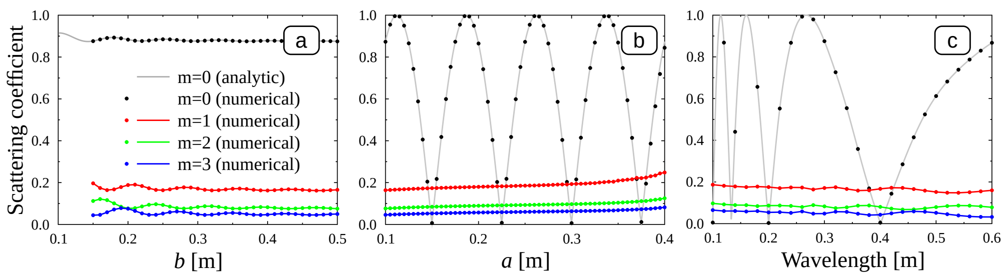

Fig. 4. Scattering coefficients in each cylindrical order as a function of b (a), a (b) or λ (c). For each parameter study, the other two parameters are fixed in the same way as described in Fig. 3 caption.

4. A comparison under plane wave incidence

Having known the inherent scattering properties, it is meaningful to verify the findings by some numerical experiments. Notice that scattering patterns obtained in such numerical experiments are source-dependent. Nevertheless they can qualitatively reflect the performances of the cloaks. Here a plane wave of unit amplitude is chosen as the incident wave in all calculations. The wave has $\lambda = 0.15$m and travels from left to right. Three types of cloaks are studied, including the previously reported simplified linear cloak, improved linear cloak, and the simplified quadratic cloak, all with a PEC lining. For each type of cloak, two cloak scales are examined. While a is kept constant at 0.1m, b is at 0.15m or 0.3m. The calculated far field scattering patterns are summarized in Fig. 5. For reference, the scattering pattern for a bare PEC cylinder with a 0.1m radius is also imposed in the panels. All scattered far fields are normalized to the maximum scattered far field by the bare PEC cylinder.

For the previously reported simplified linear cloak, the scattering remains high and varies wildly in angular direction, regardless of the cloak thickness. The sharp variation of the scattered field value along the angular direction suggests that the high-order cylindrical waves experience heavy scatterings. In comparison, the improved linear cloak causes a relatively low scattering at all cloak thicknesses. The reduction in scattering is around 10dB in the backward and forward directions as compared to the bare PEC cylinder. The relatively small angular dependence of the scattering pattern indicates that the scattering by such a cloak is dominated by the zeroth-order cylindrical wave. The quadratic cloak experiences heavy scattering when $b = 0.15$m. The scattering is almost as large as that caused by the bare PEC cylinder. This is due to the violation of the thickness constraint $b > 2a$. At $b = 0.3$m, the scattering has dropped greatly, and becomes comparable to that caused by an improved linear cloak.

#88913 - $15.00 USD Received 22 Oct 2007; revised 4 Dec 2007; accepted 5 Dec 2007; published 13 Dec 2007

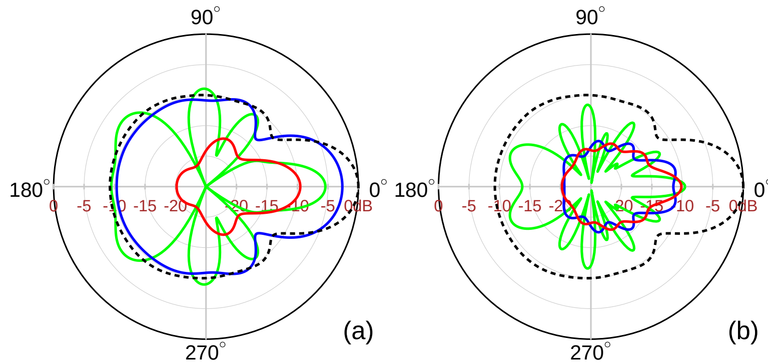

Fig. 5. The far field scattering patterns by the simplified cloaks with an identical plane wave incidence (amplitude at 1). Green curves are for the previously reported simplified linear cloak. The red curves are for the improved simplified linear cloak. The blue curves are for the simplified quadratic cloak. Dashed black curves are for the bare PEC cylinder. (a) $a = 0.1$m, $b = 0.15$m; (b) $a = 0.1$m, $b = 0.3$m.

The E_z field patterns together with the Poynting vectors for three types of cloaks, all with $a = 0.1$m and $b = 0.3$m, are shown in Fig. 6(a)-(c). In all cases a PEC lining is present. These field plots can be compared closely with those derived analytically in Fig. 1. The streamlines for poynting vectors are starting from the same positions as those shown in Fig. 1. By comparing the corresponding fields in Figs. 1 and 6, we see that all simplified cloaks cause perturbations to the plane wave. Nevertheless, all simplified cloaks are observed to imitate their respective ideal versions by bending the Poynting vectors around the inner shell and then returning the vectors back to the original trajectories. The perturbation caused by the previously reported simplified linear cloak [Fig. 6(a)] is larger than that caused by the improved linear cloak [Fig. 6(b)]. By a careful comparison of fields in Fig. 6(b) and (c), one can see that quadratic cloak [Fig. 6(c)] bends the EM field more severely at locations closer to the cloak's inner surface, whereas for the linear cloak [Fig. 6(b)] the bending happens evenly at all radial locations. This difference is also seen between the ideal fields shown in Fig. 1(b) and (c). In all cases, the maximum field amplitude has increased from the ideal case, which is -1→1. The scattered near fields (E_z) by three cloaks are plotted in Fig. 6(d)-(f). The scattered field induced by the previously reported simplified linear cloak [Fig. 6(d)] is not only very high in amplitude, but also rather inhomogeneous angularly. In comparison, the scattered fields by the improved linear cloak [Fig. 6(e)] and the simplified quadratic cloak [Fig. 6(f)] are dominated by the zero$^{\text{th}}$-order cylindrical wave with significantly lower amplitude. These scattered near fields comply well with the results we obtained in Fig. 3 and Fig. 5.

5. Discussion and conclusion

One advantage of the simplified quadratic cloak is that it can be made non-magnetic as its permitivity can be kept at constant 1 [9]. However, this advantage is valid only for normal incidence with a TM polarization (magnetic field is directed along z direction). In most real-world applications, a robust cloak needs to account for incoming EM fields in all polarizations. Hence, anisotropic and gradient permitivity profiles are still necessary for such simplified quadratic cloaks. If we compare the material parameters plotted in Figs. 2(a) and (b), we see that the simplified quandratic cloak has two gradient parameters while the improved linear cloak has only one. In addition, the variation of μ_θ for the simplified quadratic cloak is much sharper at $r \to a$. These two factors suggests that the realization of a simplified quadratic cloak is likely

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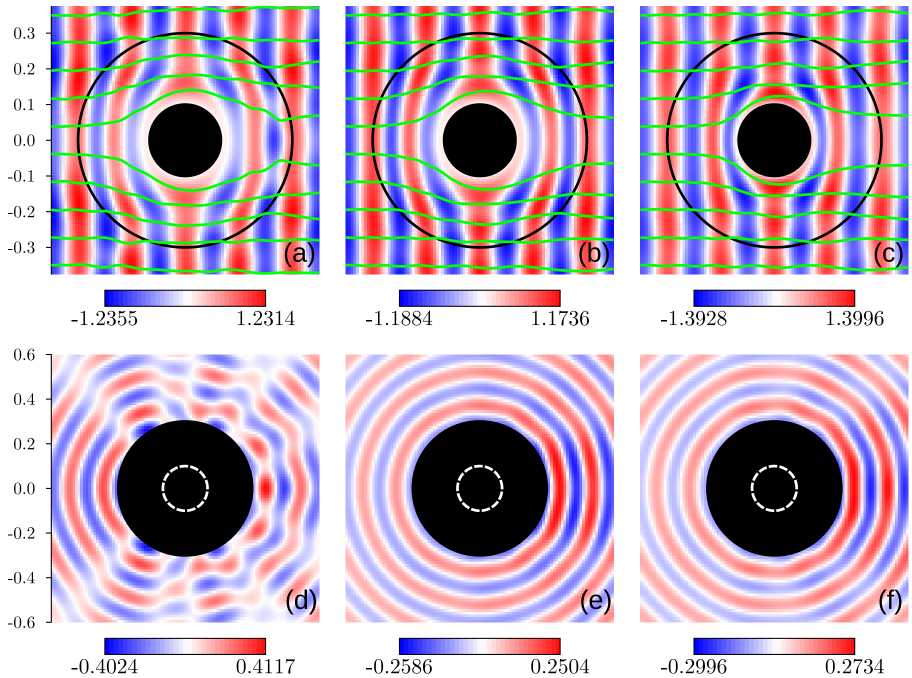

Fig. 6. Snapshots of E_z fields around (a) a previously reported simplified linear cloak, (b) an improved linear cloak, and (c) a simplified quadratic cloak. $a = 0.1$m, $b = 0.3$m. Poynting vectors are shown in green lines. The scattered E_z fields of the three cloaks in (a)-(c) are shown respectively in (d)-(f). Domain for (a)-(c): $0.75 \times 0.75\text{m}^2$, and for (d)-(f): $12 \times 12\text{m}^2$.

to be more difficult compared to the improved linear cloak.

In conclusion, we have compared the scattering characteristics of three types of simplified cylindrical cloaks, including the previously reported simplified linear cloak, the improved linear cloak, and the simplified quadratic cloak. Among the cloaks, both the improved linear cloak and the quadratic cloak are impedance-matched with the exterior domain at their outer surfaces. Therefore these two cloaks exhibit better invisibility performances as compared to the previously reported simplified linear cloak. The simplified quadratic cloak however requires a minimum thickness (outer radius is twice of inner radius) in order to achieve low-scattering operation. Hence it is not suitable for constructing thin cloaks. The scattering induced by the improved linear cloak is found to be at a steadily low level regardless of its thickness. Together with its relatively low requirement on material parameters, the improved linear cloak can potentially be a better candidate for realizing a near-invisible cloak.

Acknowledgement

This work is supported by the Swedish Foundation for Strategic Research (SSF) through the INGVAR program, the SSF Strategic Research Center in Photonics, and the Swedish Research Council (VR).