Comsol Problem Set

16 Oct 2013

Each of the following problems should be solved in a group of three students. All the problems must be solved using COMSOL Multiphysics. General procedure

1. Write down the equations and boundary conditions.
2. Use suitable scales and make the equations dimensionless.
3. Solve the equations in dimensionless for by defining appropriate parameter values and by setting some of them to unity.

1. Enzyme catalysed reactions $A \rightarrow B$ have a rate expression of the form, known as the Monod kinetics:
$$r_A = -\frac{k\,C_A}{K_m + C_A},$$
where $k,\ K_m$ are constants. One such reaction is carried out in a cylindrical region of radius R under steady state conditions, with the surface concentration $C_A = C_{A0}$ at $r = R$.

   Determine $C_A(r)$ of species A, for various values of the parameter Damköhler number $\mathrm{Da} \equiv k\,R^2/(\mathcal{D}\,C_{A0})$, and $\gamma = K_m/C_{A0}$. Compare the asymptotic expression with the numerical solution for small values of Da.

2. Transient diffusion through permeable walls of a tube. A cylindrical tube (whose radius R is small compared to its length L) with open ends is immersed and equilibriated in a fluid containing a species at a concentration C_0. At time $t = 0$ the outside (reservoir) concentration is suddenly dropped to $C_1 < C_0$. The wall is permeable to the solute, so that the solute from inside the tube diffuses out through the open ends as well as through the walls. The wall permeability (mass transfer coefficient) is k_s so that the wall flux is given by
$$j_{\mathrm{wall}} = k_s\,(C - C_1)$$
where C is the instantaneous concentration of the solute near the internal surface of the wall.

   (a) Solve for the concentration $C(r,t)$ inside the tube.

   (b) Radially integrate the concentration
$$\bar{C}(t) = \frac{2}{R^2}\int_0^R \mathrm{d}r\; C(r,t)\,r$$

(c) At some locations far away from the ends of the tube (closer to the middle), plot the dimensionless radially averaged concentration

$$\psi(t) \equiv \frac{\bar{C} - \bar{C}_1}{\bar{C}_0 - \bar{C}_1}$$

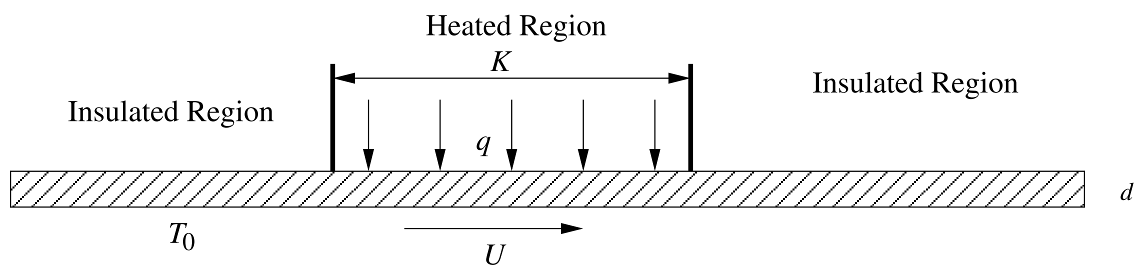
against time t. and compare it with the expression

$$\psi = e^{-2\,k_s\,t/R}$$

for small times $k_s t/R \ll 1$.

3. A pipe of diameter 1 m carries crude oil (ρ = 0.8 g/cc, μ = 3 mPa s, k = 0.5 kJ/m h K, C_p = 2 kJ/kg K) initially at 30°C, over several hundred kilometers at a rate of 5 kg/s. After a certain extent, it is taken underwater, where the surface temperature of the pipe suddenly drops to T = 4°C. Write down the dimensionless equations and boundary conditions. Setup and solve the problem in dimensionless form, and solve for the temperature profile inside the pipe. Obtain a plot of the radially averaged temperature profile as a function of the length of the pipe. From the final results reinterpret the plots in dimensional form.

4. A roll of plate of thickness d is continuously moved at a velocity U through a section of length K, which heats it up with a heat flux q_0. Before the plates enters the heating section, it is at a temperature T_0; and after it passes the heating section it passes through an insulated section with no loss of heat.



Write down the equations in dimensionless form, and solve it numerically for the dimensionless temperature

$$\Theta(x) \equiv \frac{T(x) - T_0}{qL/k}$$

Compare the temperature in the heated section with the expression

$$\Theta = \frac{1}{\mathrm{Pe}^2}\left(1 - e^{\mathrm{Pe}(\zeta-\lambda)}\right) + \frac{\zeta}{\mathrm{Pe}}$$

where $\zeta \equiv x/d$, and $\mathrm{Pe} = \frac{Ud}{\alpha}$.

5. Nusselt Number in the Entrance Region (in temperature): Consider the Graetz problem where a fully developed laminar velocity profile enters a pipe with a temperature T_0. It is subjected to a different wall temperature T_w starting at $x = 0$. Write a dimensionless governing equation for the problem and solve for the temperature profile. Plot the local Nusselt Number, defined as

$$\mathrm{Nu} \equiv \frac{hD}{k} = \frac{-\frac{\partial\Theta}{\partial n}}{\Theta_{r=0} - \Theta_w},$$

as a function of the length of the pipe (in the entrance region: meaning where the temperature inside has not reached the wall temperature). Let Nu_h be the typical Nusselt number at half way in the entrance region. Note down this value. Now change the Reynolds number Re, keeping Pe fixed. Then vary Pe. At the end get a table of values of Nu_h for Re= 0.1, 1, 10, 100. and Pe= 0.01, 0.1, 1, 10. Plot Nu_h against Re for various fixed values of Pe, and Nu_h against Pe, for various fixed values of Re, in a log-log plot. Find out the slope and hence the constants C, a and b in the expression

$$\mathrm{Nu}_h = C\,\mathrm{Re}^a\,\mathrm{Pe}^b$$

6. Thermal entrance length : Consider the Graetz problem where a fully developed laminar velocity profile enters a pipe of radius R with a temperature T_0. It is subjected to a different wall temperature T_w starting at $x = 0$. Write a dimensionless governing equation for the problem and solve for the temperature profile. Find the typical length it takes for the temperature in the pipe to reach a steady state value. Note down this length L_T. Now change the Reynolds number Re, keeping Pe fixed. Now change the Pe. At the end get a table of values of dimensionless length L_T/R for Re= 0.1, 10 and Pe= 0.01, 0.1, 1, 10, 100. Plot the dimensionless length against Pe, for various fixed values of Re, in a log-log plot. Find out the slope and hence the constants C, a, and b in the expression

$$\frac{L_T}{R} = C\,\mathrm{Re}^a\,\mathrm{Pe}^b$$

7. Nusselt Number in the fully developed Region (in temperature): Consider the Graetz problem where a fully developed laminar velocity profile enters a pipe with a temperature T_0. It is subjected to a different wall temperature T_w starting at $x = 0$. Write a dimensionless governing equation for the problem and solve for the temperature profile. Plot the local Nusselt Number, defined as

$$\mathrm{Nu} \equiv \frac{hD}{k} = \frac{-\frac{\partial\Theta}{\partial n}}{\Theta_b - \Theta_w},$$

as a function of the length of the pipe. Here,

$$\Theta_b = \frac{\int_0^R \mathrm{d}r\, u\, r\, \Theta}{\int_0^R \mathrm{d}r\, u\, r}$$

Let Nu_∞ be the typical Nusselt number which is a constant for large length. Note down this value. Now change the Reynolds number Re, keeping Pe fixed. Then vary Pe. At the end get a table of values of Nu_h for Re= 0.1, 100. and Pe= 0.01, 1, 100. What can you conclude from this table?

8. Nusselt Number in the Entrance Region (in temperature) specified heat flux: Consider the Graetz problem where a fully developed laminar velocity profile enters a pipe with a temperature T_0. It is subjected to a constant heat flux q_w from the wall. The dimensionless temperature is defined as

$$\Theta = \frac{T - T_0}{q\,R/k}$$

Write a dimensionless governing equation for the problem and solve for the temperature profile. Plot the local Nusselt Number, defined as

$$\mathrm{Nu} \equiv \frac{hD}{k} = \frac{-\frac{\partial\Theta}{\partial n}}{\Theta_{r=0} - \Theta_w},$$

as a function of the length of the pipe (in the entrance region: meaning where the temperature inside has not reached the wall temperature). Let Nu_h be the typical Nusselt number at half way in the entrance region. Note down this value. Now change the Reynolds number Re, keeping Pe fixed. Then vary Pe. At the end get a table of values of Nu_h for Re= 0.1, 1, 10, 100. and Pe= 0.01, 0.1, 1, 10. Plot Nu_h against Re for various fixed values of Pe, and Nu_h against Pe, for various fixed values of Re, in a log-log plot. Find out the slope and hence the constants C, a and b in the expression

$$\mathrm{Nu}_h = C\,\mathrm{Re}^a\,\mathrm{Pe}^b$$

9. Thermal entrance length (constant heat flux): Consider the Graetz problem where a fully developed laminar velocity profile enters a pipe of radius R with a temperature T_0. It is subjected to a constant heat flux q_w from the wall. The dimensionless temperature is defined as

$$\Theta = \frac{T - T_0}{q R/k}$$

Write a dimensionless governing equation for the problem and solve for the temperature profile. Find the typical length it takes for the temperature in the pipe to reach a steady state value. Note down this length L_T. Now change the Reynolds number Re, keeping Pe fixed. Now change the Pe. At the end get a table of values of dimensionless length L_T/R for Re= 0.1, 10 and Pe= 0.01, 0.1, 1, 10, 100. Plot the dimensionless length against Pe, for various fixed values of Re, in a log-log plot. Find out the slope and hence the constants C, a, and b in the expression

$$\frac{L_T}{R} = C\,\mathrm{Re}^a\,\mathrm{Pe}^b$$

10. Nusselt Number in the fully developed Region (in temperature) with specified heat flux: Consider the Graetz problem where a fully developed laminar velocity profile enters a pipe with a temperature T_0. It is subjected to a constant heat flux q_w from the wall. The dimensionless temperature is defined as

$$\Theta = \frac{T - T_0}{q R/k}$$

Write a dimensionless governing equation for the problem and solve for the temperature profile. Plot the local Nusselt Number, defined as

$$\mathrm{Nu} \equiv \frac{hD}{k} = \frac{-\frac{\partial\Theta}{\partial n}}{\Theta_b - \Theta_w},$$

as a function of the length of the pipe. Here,

$$\Theta_b = \frac{\int_0^R \mathrm{d}r\, u\, r\, \Theta}{\int_0^R \mathrm{d}r\, u\, r}$$

Let Nu_∞ be the typical Nusselt number which is a constant for large length. Note down this value. Now change the Reynolds number Re, keeping Pe fixed. Then vary Pe. At the end get a table of values of Nu_h for Re= 0.1, 100. and Pe= 0.01, 1, 100. What can you conclude from this table?

11. Consider a fully developed flow between two parallel plates with a constant heat flux at the plates. The spacing between the plates is $2H$. Find out the Nusselt number defined as

$$\mathrm{Nu} \equiv \frac{2hH}{k} = \frac{-\frac{\partial \Theta}{\partial n}}{\Theta_b - \Theta_w},$$

Here,

$$\Theta_b = \frac{\int_{-H}^{H} \mathrm{d}y\, u\,\Theta}{\int_{-H}^{H} \mathrm{d}y\, u}.$$

Plot the Nusselt number as a function of the dimensionless axial distance along the flow, for Re= 0.1, 100, and Pe= 0.1 and 100.

12. Consider a fully developed flow between two parallel plates with constant temperature at the walls. The spacing between the plates is $2H$. Find out the Nusselt number defined as

$$\mathrm{Nu} \equiv \frac{2hH}{k} = \frac{-\frac{\partial \Theta}{\partial n}}{\Theta_b - \Theta_w},$$

Here,

$$\Theta_b = \frac{\int_{-H}^{H} \mathrm{d}y\, u\,\Theta}{\int_{-H}^{H} \mathrm{d}y\, u}.$$

Plot the Nusselt number as a function of the dimensionless axial distance along the flow, for Re= 0.1, 100, and Pe= 0.1 and 100.

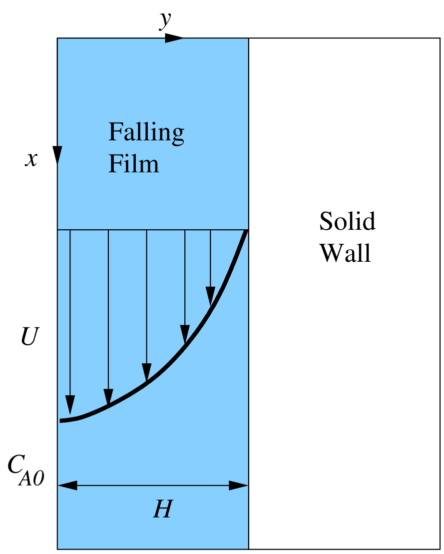
13. A liquid film of thickness H flows down a vertical impermeable solid surface (as shown below). The flow is laminar and fully developed. The other side (left) of the film is in contact with air which has a species A that is to be removed. Take the value of Peclet number Pe = 100. The species A dissolves in the liquid and undergoes a irreversible first order reaction. The concentration of the species at the air-liquid interface is C_{A0}. Write the dimensionless concentration equations, and solve for the concentration profile. Find the local Sherwood number Sh for the species, defined as

$$\mathrm{Sh} \equiv \frac{k_c D}{k} = \frac{-\frac{\partial X_A}{\partial n}}{X_b - X_0},$$

as a function of the length of the pipe. Here,

$$X_b = \frac{\int_{0}^{H} \mathrm{d}y\, u\,X}{\int_{0}^{H} \mathrm{d}y\, u}.$$

Plot the values of Sh for Damkohler number Da= 0.01, 0.03, 0.1, 0.3.



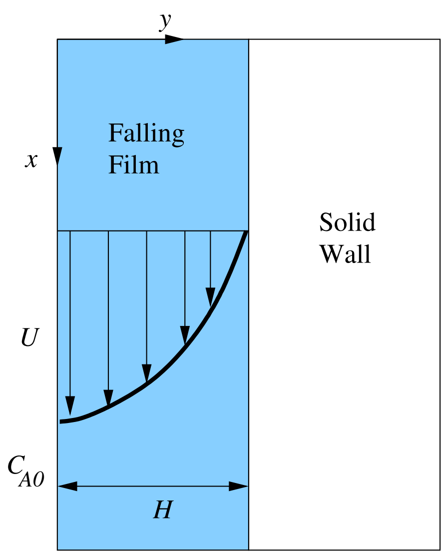
14. A liquid film of thickness H flows down a vertical impermeable solid surface (as shown below). The flow is laminar and fully developed. The other side (left) of the film is in contact with air which has a species A that is to be removed. Take the value of Peclet number Pe = 100. The species A dissolves in the liquid and undergoes a irreversible first order reaction. The concentration of the species at the air-liquid interface is C_{A0}. Write the dimensionless concentration equations, and solve for the concentration profile. Find the local Sherwood number Sh for the species, defined as

$$\mathrm{Sh} \equiv \frac{k_c D}{k} = \frac{-\frac{\partial X_A}{\partial n}}{X_b - X_0},$$

as a function of the length of the pipe. Here,

$$X_b = \frac{\int_0^H \mathrm{d}y\; u\, X}{\int_0^H \mathrm{d}y\; u}.$$

Plot the values of Sh for Damkohler number Da= 10, 30, 100, 300.



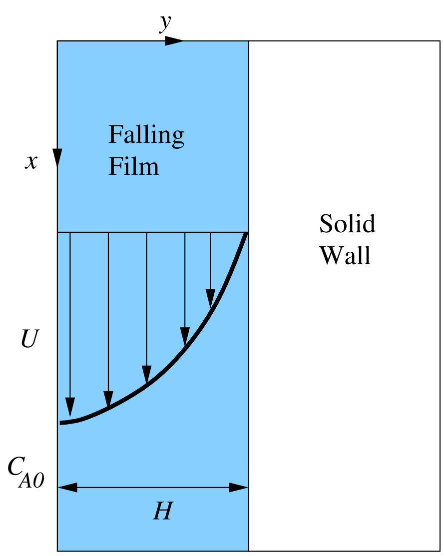
15. A liquid film of thickness H flows down a vertical impermeable solid surface (as shown below). The flow is laminar and fully developed. The other side (left) of the film is in contact with air which has a species A that is to be removed, which dissolves in the liquid (without any reaction). The concentration of the species at the air-liquid interface is C_{A0}. Write the dimensionless concentration equations, and solve for the concentration profile. Find the local Sherwood number Sh for the species, defined as

$$\mathrm{Sh} \equiv \frac{k_c D}{k} = \frac{-\frac{\partial X_A}{\partial n}}{X_b - X_0},$$

as a function of the length of the pipe. Here,

$$X_b = \frac{\int_0^H \mathrm{d}y\ u\, X}{\int_0^H \mathrm{d}y\ u}.$$

Plot the values of Sh as a function of the dimensionless length in the developing region for Peclet number Pe= 0.1, 1, 10.



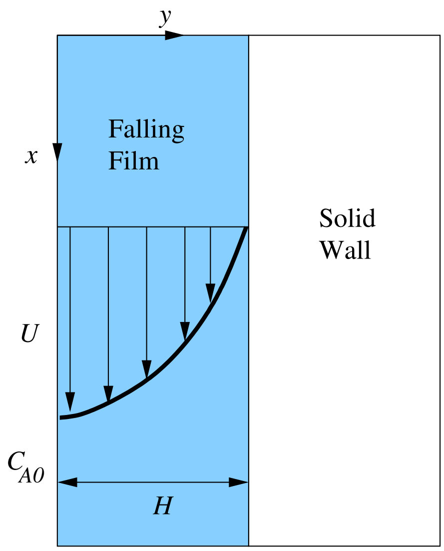
16. A liquid film of thickness H flows down a vertical impermeable solid surface (as shown below). The flow is laminar and fully developed. The other side (left) of the film is in contact with air which has a species A that is to be removed, which dissolves in the liquid (without any reaction). The concentration of the species at the air-liquid interface is C_{A0}. Write the dimensionless concentration equations, and solve for the concentration profile. Find the local Sherwood number Sh for the species, defined as

$$\mathrm{Sh} \equiv \frac{k_c D}{k} = \frac{-\frac{\partial X_A}{\partial n}}{X_b - X_0},$$

as a function of the length of the pipe. Here,

$$X_b = \frac{\int_0^H \mathrm{d}y\ u\, X}{\int_0^H \mathrm{d}y\ u}.$$

Plot the values of Sh as a function of the dimensionless length for Peclet number Pe= 0.1, 1, 10.



17. Diffusion in a sphere with fast reaction. Consider a spherical catalyst pellet immersed in a fluid. The concentration of a species in the fluid is constant C_{A0}. Inside the pellet the species undergoes a irreversible first order reaction. Set up the problem in a dimensionless form. Find the radial concentration profile for various values of the Damkohler number Da= 10, 100.

18. Cross flow filtration. Consider a flow between two porous plates. In the x direction the flow is driven by a pressure gradient G. In the vertical y direction there is flow with a constant velocity V (inlet and outlet), thereby generating a cross-flow between the plates. Obtain the horizontal velocity profiles for Re $\ll 1$ and Re $\gg 1$ (Take any two sample values in the low and high Re limit). Compare profile with the exact solution to the equation

$$\rho V \frac{\mathrm{d}u}{\mathrm{d}y} = G + \mu \frac{\mathrm{d}^2 u}{\mathrm{d}y^2}.$$