MODELLING HYSTERESIS LOOPS OF SOFT FERRITE MATERIALS

Mauricio Esguerra
EPCOS AG, Ferrites Division, Product Design Department
P. O. Box 801709, D-81617 Munich, GERMANY

Abstract – An extension to Hodgdon´s hysteresis equation and its solution for minor loops in terms of measured major loops is presented. Various application relevant parameters for soft ferrite materials are derived and the results compared to well-known models for the small signal regime.

1. Hodgdon´s Hysteresis Model

Many attempts have been done in the past to quantitatively describe hysteresis loops. One of the most promising approaches is the description of the memory-effect inherent to hysteresis by means of differential equations [1,2]. The one formulated by Hodgdon with the B-field as dependent variable,

$$\frac{dH}{dB} = \alpha \cdot \operatorname{sgn}\left(\frac{dB}{dt}\right)\left(f(B) - H\right) + g(B) \quad (1)$$

has two major advantages:

- direct integrability
- applicable to most practical problems, where voltage (and consequently flux density) rather than current is the known signal

Hodgdon´s own solution starts from a parametrization of the curve, whereas the author's approach is based on regarding the major loop as the particular solution of the equation [3]. From there all other solutions, e.g. minor loops, can be obtained without explicitly considering the material functions *f(B)* and *g(B)*. The upper $H'_U(B)$ and lower $H'_L(B)$ minor loop branches can be therefore expressed in terms of the corresponding measured major loop branches $H_U(B)$ and $H_L(B)$.

The aim of the present work is to improve two major aspects. On the one hand calculated minor loops close to saturation deviate systematically from measured ones. On the other hand the quality and/or number of points of the measured major loops is often not enough for an accurate minor loop calculation.

2. Model's Extension

In order to remedy the first deficiency, the differential equation has been modified by regarding the parameter α as a function of *B* rather than of the amplitude $\hat{B}$ [3], the equation being still integrable. From the solution for symmetric minor loops given in [3], eq. (3) follows for the integral over $\alpha(B)$:

$$e^{\int_{-\hat{B}}^{\hat{B}} \alpha(B')dB'} = \frac{H_U(\hat{B}) - \hat{H}(\hat{B})}{\hat{H}(\hat{B}) - H_L(\hat{B})} \quad (2)$$

According to the symmetry of eq. (1) the function $\alpha(B)$ is even, such that the general integral is given by:

$$C(B_1, B_2) \equiv e^{\int_{B_1}^{B_2} \alpha(B')dB'} = \sqrt{\frac{H_U(B_2) - \hat{H}(B_2)}{\hat{H}(B_2) - H_L(B_2)} \cdot \frac{\hat{H}(B_1) - H_L(B_1)}{H_U(B_1) - \hat{H}(B_1)}} \quad (3)$$

The commutation curve needs also to be modified with respect to [3], eq. (5) in order to account for the above mentioned improvement of the saturation behavior. The following *ansatz* takes this into consideration:

$$\hat{H}\left(\hat{B}\right) = H_L\left(\hat{B}\right) - H_c\left(1 - \tfrac{\hat{B}}{B_s}\right)^{\xi} \quad (4)$$

The exponent ξ can be determined from the slope of the commutation curve around the origin:

$$\xi = \alpha_0 B_s; \quad \alpha_0 = \frac{1}{\mu_o H_c}\left(\frac{1}{\mu_i} - \frac{1}{\mu_c}\right) \quad (5)$$

The solution for a (not necessarily symmetric) minor loop reads finally:

$$H'_L(B) = H_L(B) + \frac{H_m - H_L(B_m)}{C(B_m, B)} \quad (6a)$$

$$H'_U(B) = H_U(B) + \frac{H_M - H_U(B_M)}{C(B, B_M)} \quad (6b)$$

The extreme field strength points H_m and H_M are given by

$$H_m(B_m, B_M) = \frac{H_U(B_m)\cdot C(B_m, B_M) - \frac{H_L(B_m)}{C(B_m, B_M)} + H_L(B_M) - H_U(B_M)}{C(B_m, B_M) - \frac{1}{C(B_m, B_M)}} \quad (7a)$$

$$H_M(B_m, B_M) = -H_m(-B_M, -B_m) \quad (7b)$$

3. Major Loop Fit Functions

To improve accuracy, empirical fit functions for the lower and the upper branches of the major loop have been found as an extension of the “linear” approximation given by eq. 8 in [3]:

$$H_L(B) = \frac{B}{\mu_o \mu_c} \cdot \frac{1}{1 - \left(\frac{B}{B_s}\right)^a} + H_c \quad (8a)$$

$$H_U(B) = \frac{B}{\mu_o \mu_c} \cdot \frac{1}{1 - \left(\frac{B}{B_s}\right)^b} - H_c \quad (8b)$$

The parameters H_c (coercive field), μ_c slope at H_c) and B_s (saturation flux density) have an obvious physical meaning, whereas *a* and *b* describe the squareness of the loop. Fig. 1 shows an example for Material N87 at 100℃ with following parameters: μ_i= 3980, B_s=0.398 T, *a*=4.91, *b*=4.32, H_c=11.9 A/m, μ_c=5148. The fit curves (lines) and the measurement (points) show an excellent agreement.

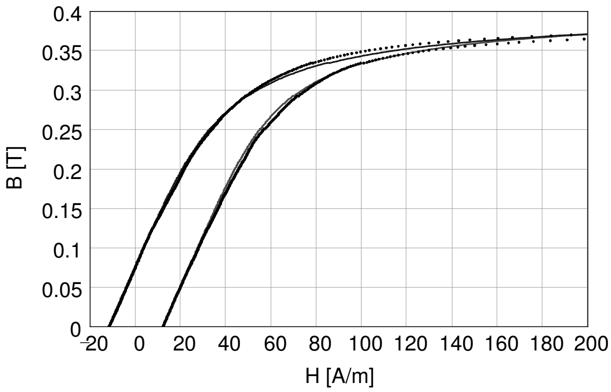

Fig. 1: Major hysteresis loop´s fit functions

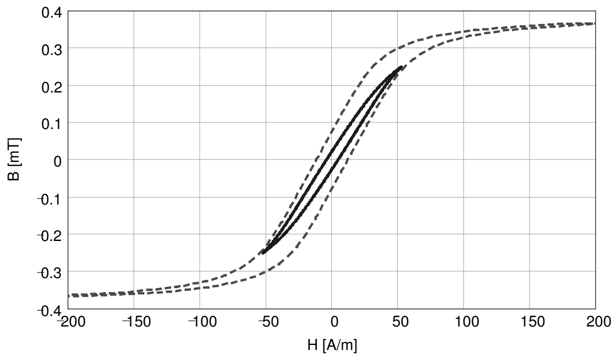

Fig. 2: Symmetric Minor Loop with $-B_m=B_M$=0.25 T (solid line) with major loop (dotted line)

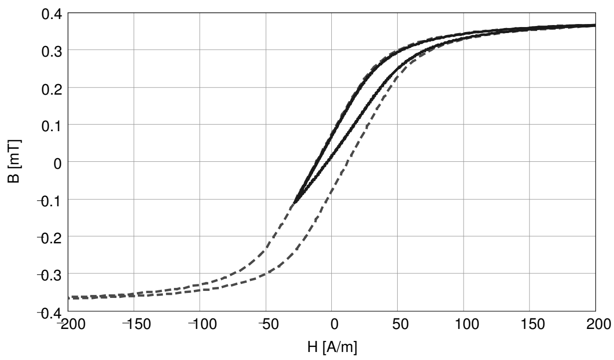

Fig. 3: Non-Symmetric Minor Loop with B_m=-0.11 T and B_M=0.39 T (solid line) with major loop (dotted line)

Figs. 2 and 3 show a symmetric and a non-symmetric minor loops with ΔB=0.5 T for N87 at 100℃. With the present model it is possible to yield good agreement not only in the symmetric case, but especially in the way the minor upper branch follows the major branch close to saturation.

4. Derived Parameters

The main purpose of hysteresis modeling is the evaluation of core losses. However, the calculation of the enclosed area of a minor loop cannot be evaluated by a closed integral. In the approximation of symmetric, small excitations the losses are given by:

$$w_s = 2\int_0^{\hat{B}} \left[H_L'(B) - H_U'(B)\right] dB \approx \frac{4}{3} H_c \alpha_0^{\,2} \hat{B}^3 \qquad (9)$$

This result coincides with the previous derivation [3] and is in accordance with Rayleigh´s law [4].

The hysteresis material constant (in the limit $\hat{B} \to 0$) is related to the harmonic distortion and reads:

$$\eta_B = \frac{4}{3\pi\mu_0 H_c}\left(\frac{1}{\mu_i} - \frac{1}{\mu_c}\right)^2 \qquad (10)$$

Both the core losses and η_B increase with the difference between the inverse values of μ_i and μ_c , i. e. the “real” and the “ideal” permeabilities [3].

The reversible permeability under dc-bias can be given in the approximation $a \approx b$ and $\hat{B} \to 0$ by:

$$\mu_{rev}(B_{dc}) = \left(\frac{1 + (a-1)\cdot\left(\frac{B_{dc}}{B_s}\right)^a}{\left[1 - \left(\frac{B_{dc}}{B_s}\right)^a\right]^2} \cdot \frac{1}{\mu_c} + \frac{1}{\left(1 - \frac{B_{dc}}{B_s}\right)\cdot\left[2 - \left(1 - \frac{B_{dc}}{B_s}\right)^{\alpha_0 \cdot B_s}\right]} \cdot \left(\frac{1}{\mu_i} - \frac{1}{\mu_c}\right) \right)^{-1} \qquad (11)$$

This equation shows that the reversible permeability depends from two terms representing both loss-independent (squareness parameter *a*) and loss-dependent ($1/\mu_i - 1/\mu_c$) contributions. Thus, improving the loss quality of a material does not necessarily yield an improvement of its dc-bias behavior.

Fig. 4 shows the normalized reversible permeability as a function of dc-flux density referred to saturation for material T55 at 25°C. For reference the universal curve after Gans [4] is also shown with an excellent agreement.

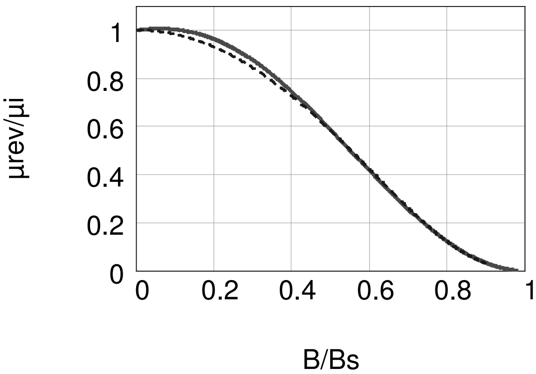

Fig. 4: Calculated normalized reversible permeability vs. flux density (solid line) in comparison to the Gans curve (dotted line)

References

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