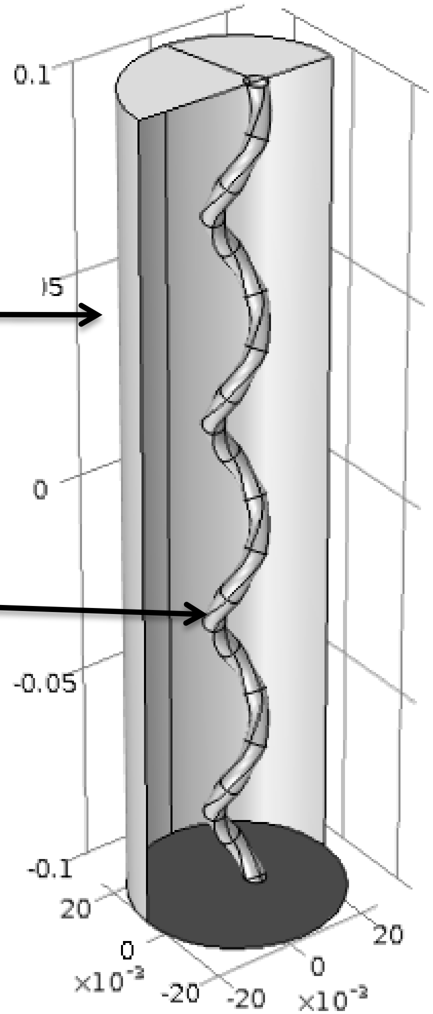
Corona Discharge problem

Collecting electrode

Emitter electrode



Electrostatics with potential ϕ:

$$\nabla^2 \phi = -\frac{\rho}{\varepsilon_0}$$

Charge transport ρ:

$$\nabla \cdot \left(-\rho \nabla(\phi)\right) = 0$$

Imposed potential ϕ:

$$\phi\big|_{emitter} = \phi_0$$

$$\phi\big|_{collector} = 0$$

Peek's boundary condition:

$$\mathbf{n} \cdot \nabla \phi = E_0$$

Coupled physics

This is a 3rd order equation, with 3 boundary conditions

(6) $\nabla \cdot \left(-\nabla^2(\phi)\nabla(\phi)\right) = 0$

At the emitter :

(7) ϕ = V0

at the collector (ground) :

(8) ϕ = 0

The third boundary condition is given by Peek's formula, which determines the value of the electric field at the emitter electrode :

(9) $\mathbf{n} \cdot \nabla \phi = E_0$

The problem is well-posed. However under this formulation, it turns out to be very difficult to solve.

Litterature

This system of equation (6)-(9) has been solved in the past with several methods and using different kind of algorithms. See for instance reference [1]. All of them are based on a splitting between ϕ and ρ, with an iteration scheme in order to reach convergence. None of them is based on a fully coupled scheme.

[1] S. Cristina, G. Dinelli, M. Feliziani, Numerical computation of corona space charge and V-I Characteristic in electrostatic precipitators, IEEE Trans. Ind. Appl. 27 (1) 147-153 (1991).

COMSOL Multiphysics allows for a fully coupled solution!

Rewrite the problem

The initial problem is easily transformed into 2 equations, one for ϕ, the other for ρ :

(10) $$\nabla^2\phi = -\frac{\rho}{\varepsilon_0}$$

(11) $$0 = \nabla(\phi)\cdot\nabla(\rho) - \frac{\rho^2}{\varepsilon_0}$$

with boundary conditions :

(7) ϕ = V0

(8) ϕ = 0

(9) $\mathbf{n}\cdot\nabla\phi = E_0$

The equation for ϕ requires 2 boundary conditions (8) and (9).

The equation for ρ is a first order equation and requires only one boundary condition.

The problem is that there is no boundary condition for ρ : its value at the emitter is unknown and there is no simple way to determine it (apart from more elaborate plasma modeling). Instead, boundary condition (7) should be used, but how?

New equation for ρ

In order to circumvent this problem, simply split ρ into 2 parts, the first one being constant and the other one being space-dependent :

(12) ρ = ρ0p + dρ

ρ0p is constant in space

dρ is space dependent

<u>Suppose ρ0p known</u>

The equation is now set for dρ:

(14) $$0 = \nabla(\phi)\cdot\nabla(d\rho) - \frac{(\rho 0 p + d\rho)^2}{\varepsilon_0}$$

With one very simple boundary condition :

(15) dρ = 0 at the emitter electrode

Equation for ϕ

the equation for PHI is unchanged :

(13) $$\nabla^2\phi = -\frac{(\rho 0 p + d\rho)}{\varepsilon_0}$$

with boundary conditions :

(8) ϕ=0

(9) $\mathbf{n}\cdot\nabla\phi = E_0$

Fix the constant $\rho 0$

Remains the constant value at the emitter to be determined by a Lagrange multiplier $\rho 0$:

at one (arbitrary) point of the emitter P, COMSOL easily enforces the boundary condition (7) :

(7) $0 = V0-\phi$

through a Lagrange multiplier $\rho 0$ that is integrated over the point P.

This equation is easily set-up with a “weak form, point” application mode.

The contribution to the weak form equation is set as (after integration by part) :

(16) rho0_test*(V0-PHI)

at one point P located on the emitter.

COMSOL

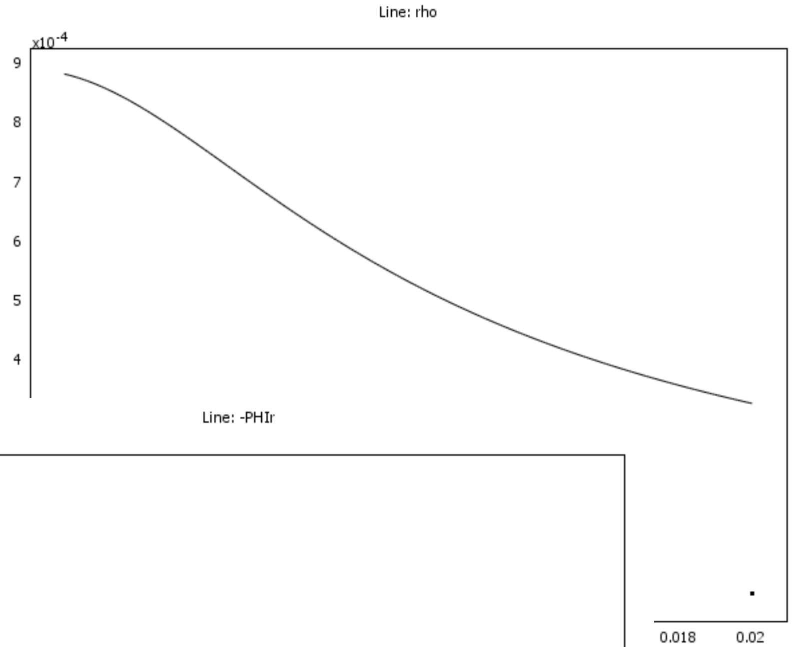
Remarks

Because $\rho 0$ is only defined in point P, we need to use a so-called "integration coupling variable" which makes the source variable $\rho 0$ available everywhere in the subdomain. The name of this new variable is defined as $\rho 0p$. That explains why we use $\rho 0p$ and not $\rho 0$ as constant value in the definition of ρ (equation (12)).

It is possible to solve the fully coupled problem in one single step if the initial condition for ϕ is set-up so that it is consistent with the boundary conditions (don't start from 0 as initial condition, since 0 is a trivial solution of the problem).

The most accurate way to find a good initial condition for ϕ is to solve first for ϕ. Solve for all 3 equations with ϕ as initial condition by clicking the restart button. Use the Solver Sequence in order to automatically store this 2-steps procedure.

1D axi or 2D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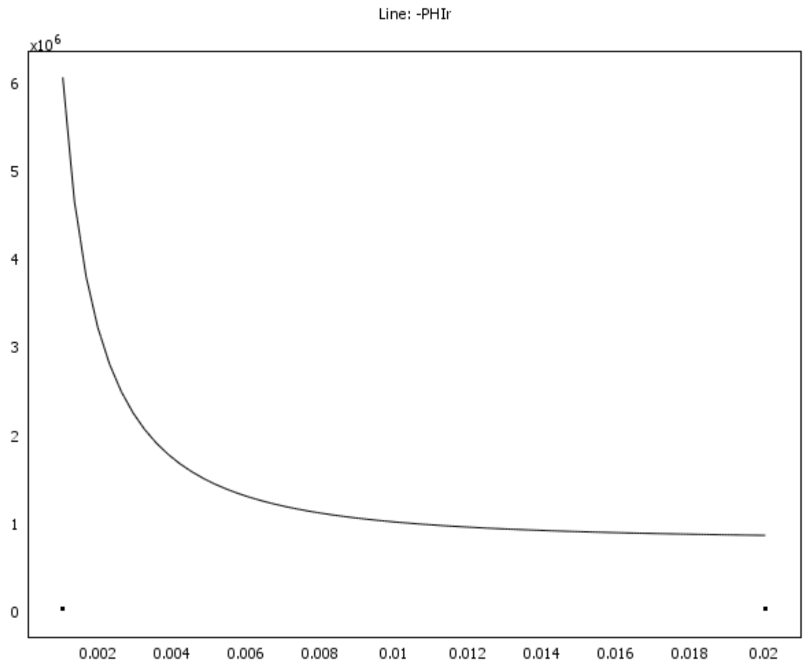
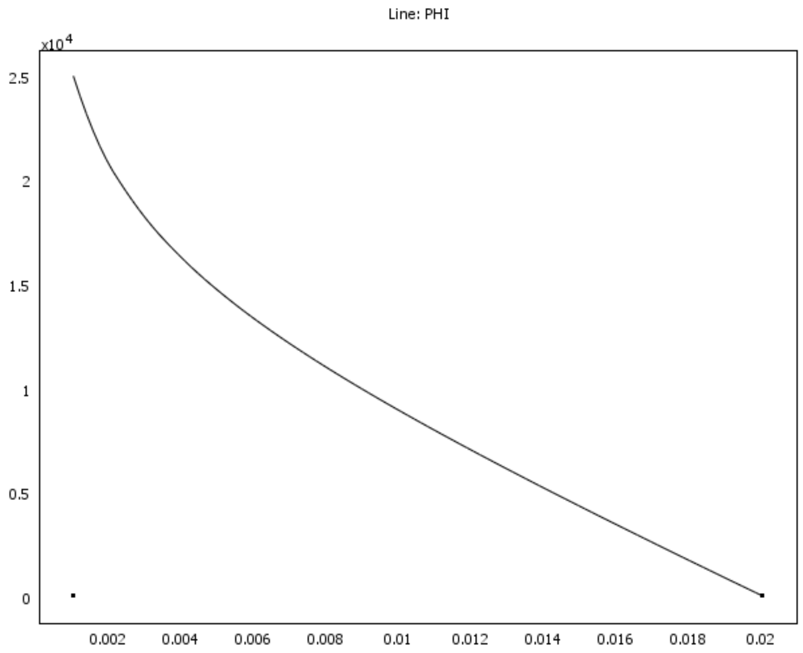
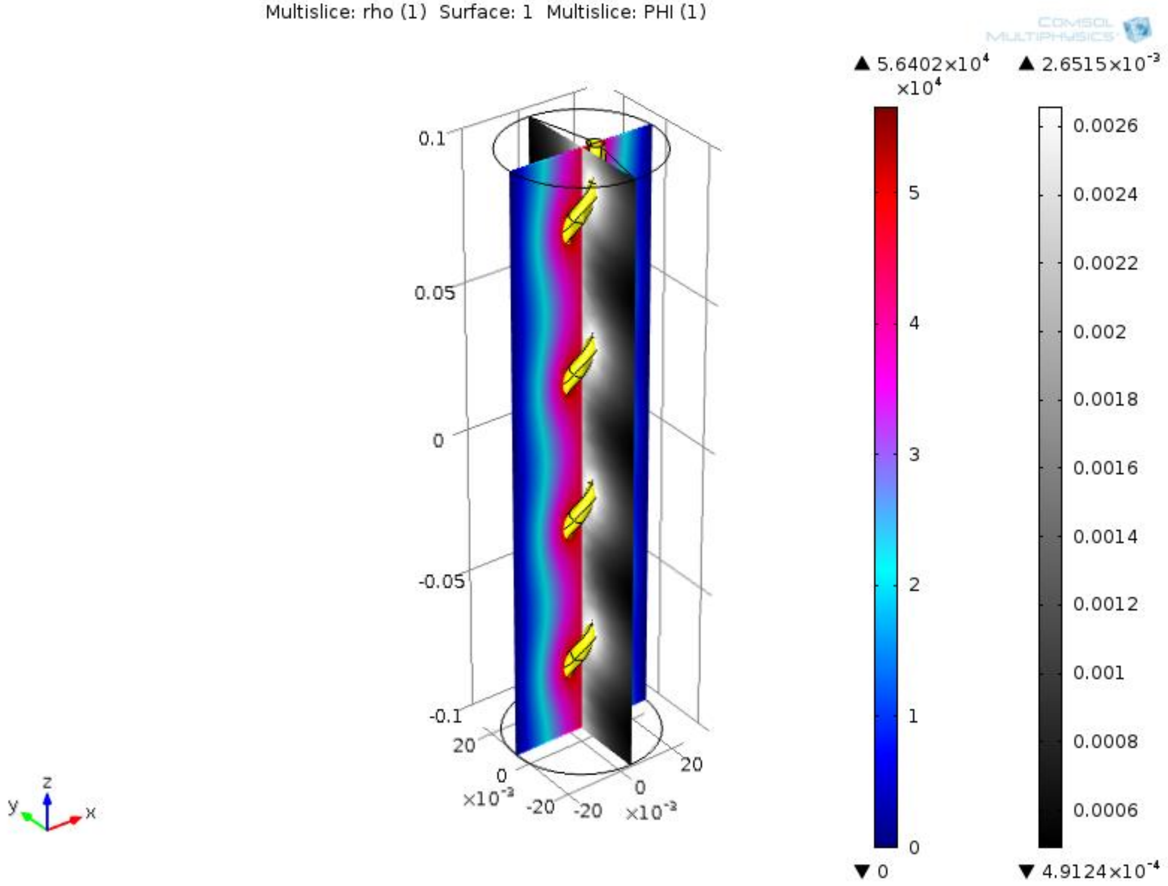
x10^-4
9
8
7
6
5
4
0.018
0.02


Line: -PHIr
x10^6
6
5
4
3
2
1
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02


Line: PHI
x10^4
2.5
2
1.5
1
0.5
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02

Full 3D



Multislice: rho (1) Surface: 1 Multislice: PHI (1)
▲ 5.6402×10^4
×10^4
▲ 2.6515×10^-3
0.1
0.05
0
-0.05
-0.1
20
0
×10^-3
-20
-20
0
×10^-3
20
5
4
3
2
1
0
0.0026
0.0024
0.0022
0.002
0.0018
0.0016
0.0014
0.0012
0.001
0.0008
0.0006
▼ 0
▼ 4.9124×10^-4
z
y
x